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Index reduction of differential algebraic equations by differential Dixon resultant☆

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ABSTRACT

High index differential algebraic equations (DAEs) are ordinary differential equations (ODEs) with constraints and arise frequently from many mathematical models of physical phenomenons and engineering fields. In this paper, we generalize the idea of differential elimination with Dixon resultant to polynomially nonlinear DAEs. We propose a new algorithm for index reduction of DAEs and establish the notion of differential Dixon resultant, which can provide the differential resultant of the enlarged system of original equations. To make use of structure of DAEs, variable pencil technique is given to determine the termination of differentiation. Moreover, we also provide a heuristic method for removing the extraneous factors from differential resultant. The experimentation shows that the proposed algorithm outperforms existing ones for many examples taken from the literature.

1. Introduction

Modeling with differential algebraic equations (DAEs) plays a vital role in a variety of applications [16], for constrained mechanical systems, control theory, electrical circuits and chemical reaction kinetics, and many other areas. In general, it is directly numerical computations difficult to solve the system of DAEs. Therefore, index reduction techniques may be necessary to get a solution [1]. The index of DAEs is a measure of the number of times needed to differentiate it to get its equivalent low index or ordinary differential equations (ODEs). There are many different index concepts for the specific DAEs, such as the differentiation index [1,3], perturbation index [3,12], tractability index [19], structural index [23], and Kronecker index [31]. There has been considerable research for the general linear and low index DAEs [16,19,21,31]. In particular, it may only solve some special DAEs by the directly numerical methods [8,18]. It is more difficult to solve the system of high index nonlinear DAEs [1–3,11,20,23].

Index reduction in the pre-analysis of DAEs solving is an active technique of research. It is equivalent to applying a sequence of differentiations and eliminations to an input system of DAEs. In [21], Pantelides gave a systematic way to reduce the high index DAEs to low index one, by selectively adding differentiated forms of the equations already appear in the system. However, the algorithm can succeed yet not correctly in some instances and be just first order [27]. Campbell's

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derivative array theory [2] needs to be computationally expensive especially for computing the singular value decomposition of the Jacobian of the derivative array equations using nonlinear singular least squares methods. In [20], Mattsson and Soderlind proposed the dummy derivative method based on Pantelides' algorithm [21] for index reduction, which is an algebraic viewpoint. In [23], Pryce proposed the signature matrix method (also called Σ-method), which can be viewed as an extension of Pantelides' algorithm [21] for any order. Recently, Wu et al. [32] generalized the Σ-method for DAEs to partial differential algebraic equations with constraints (PDAEs). After that, Qin et al. [24] generalized the Σ-method for large sacle system of DAEs. But the Σ-method relies heavily. on square (i.e. the same number of DAEs and dependent variables) and sparsity structure, which is confronted with the same drawback that can succeed yet not correctly in some DAEs arising from the specific types.

In this paper, we propose an efficient differential elimination approach for index reduction of DAEs that extends the direct elimination treatment of Yang et al. [34]. From an algebraic standpoint, differential elimination algorithms which are key for simplifying systems of polynomially differential equations and computing formal power series solutions for them. The underlying theory is the differential algebra of Ritt [28] and Kolchin [15]. Differential elimination algorithm in algebraic elimination theory is an active field and powerful tools with many important applications [7,10,26,29,34,35]. Almost all of the authors focus on the differential elimination theory for ODEs. Only Reid et al. presented an effective algorithm for computing the index of polynomially nonlinear DAE and a framework for the algorithmic analysis of perturbed system of PDAEs. This underlies the jet space approach based on differential geometry.

In this paper, we consider promoting the efficient differential elimination algorithm as natural generalization of DAEs, which is a direct and elementary approach. In particular, differential elimination with Dixon resultant can be solved by eliminating several variables at a time, simplifying the system with respect to its constraints, or determining its singular cases [34]. We can directly transform the system of DAEs to its equivalent ODEs by differential Dixon resultant. It is to obtain each eliminated equation, which just contains a dependent variable and its derivative. Differential Dixon resultant is to apply a finite number of differentiations and eliminations to uncover all hidden constraints of system of DAEs. We define a new minimum differentiation time, which is a weak differentiation index for DAEs/ODEs. It can be used as a unified formulation of differentiation times for differential elimination of DAEs/ODEs. Meanwhile, we provide a new index reduction with variable pencil and the notion of differential Dixon resultant, which is the ODEs with variables. In order to overcome the drawback of factoring a large polynomial system [34], we consider a heuristic method for removing the extraneous factors from the differential Dixon resultant matrix. Our algorithm is also suitable for the non-square nonlinear DAEs/ODEs. To the best of our knowledge, it is the first time that the generalized Dixon resultant formulation has been directly extended to the system of DAEs.

The rest of the paper is organized as follows. Section 2 gives a brief description of the generalized Dixon resultant formulation, and analyzes the size of Dixon matrix and the complexity of computing the entries of Dixon matrix. Section 3 proposes a new index reduction procedure for the system of DAEs and defines the weak differentiation index. Section 4 provides the differential elimination algorithm and some basic properties of differential Dixon resultant. Section 5 presents some specific examples in detail and comparisons of our algorithm for the system of ODEs. The final section concludes this paper.

2. Generalized Dixon resultant formulation

Following Kapur et al. [4,5,13,14,36,37], we introduce the concept of generalized Dixon resultant formulation and its properties. This technique will play a central role in our subsequent analysis. Let $X = \{x_1, x_2, \dots, x_n\}$ and $\bar{X} = \{\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n\}$ be two sets of n variables, respectively. Let $\mathbb{Q}$ be the rational field. The determinant of a square matrix A is denoted by $det(A)$.

Definition 2.1. Let $\mathcal{F} = \{f_1, f_2, \dots, f_{n+1}\} \subset \mathbb{Q}[X]$ be a set of $n+1$ polynomials in n variables. The cancellation matrix $\mathcal{C}_{\mathcal{F}}$ of $\mathcal{F}$ is the $(n+1)\times(n+1)$ matrix as follows:

$$\mathcal{C}_{\mathcal{F}} = \begin{bmatrix} f_1(x_1, x_2, \dots, x_n) & \cdots & f_{n+1}(x_1, x_2, \dots, x_n) \\ f_1(\bar{x}_1, x_2, \dots, x_n) & \cdots & f_{n+1}(\bar{x}_1, x_2, \dots, x_n) \\ f_1(\bar{x}_1, \bar{x}_2, \dots, x_n) & \cdots & f_{n+1}(\bar{x}_1, \bar{x}_2, \dots, x_n) \\ \vdots & \vdots & \vdots \\ f_1(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n) & \cdots & f_{n+1}(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n) \end{bmatrix},$$

where $f_i(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_k, x_{k+1}, x_{k+2}, \dots, x_n)$ stands for uniformly replacing x_j by $\bar{x}_j$ for all $1 \le j \le k \le n$ in f_i. The Dixon polynomial of $\mathcal{F}$ is denoted by $\theta_{\mathcal{F}} \in \mathbb{Q}[X, \bar{X}]$,

$$\theta_{\mathcal{F}} = \frac{det(\mathcal{C}_{\mathcal{F}})}{\prod_{i=1}^{n}(x_i - \bar{x}_i)}, \quad (1)$$

the row vector of Dixon derived polynomials of $\mathcal{F}$ is denoted by $P_{\mathcal{F}}$, and the Dixon matrix of $\mathcal{F}$ is denoted by $D_{\mathcal{F}}$ as follows,

$$\theta_{\mathcal{F}} = P_{\mathcal{F}} V_{\bar{X}}(\theta_{\mathcal{F}}) = V_X(\theta_{\mathcal{F}}) D_{\mathcal{F}} V_{\bar{X}}(\theta_{\mathcal{F}}), \quad (2)$$