Applied Mathematics and Computation 327 (2018) 8–21

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Wavelet Galerkin method for fourth order linear and nonlinear differential equations

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ARTICLE INFO

MSC:
65T60
65C30
31A30

Keywords:
Daubechies wavelet
2-term connection coefficients
Periodic scaling function
Multiresolution analysis
Wavelet Galerkin method

ABSTRACT

In this paper, we propose a wavelet Galerkin method for fourth order linear and nonlinear differential equations using compactly supported Daubechies wavelets. 2-term connection coefficients have been effectively used for a computationally economical evaluation of higher order derivatives. The orthogonality and compact support properties of basis functions lead to highly sparse linear systems. The quasilinearization strategy is effectively employed in dealing with wavelet coefficients of nonlinear problems. The stability and the convergence analysis, in the form of error analysis, have been carried out. An efficient compression algorithm is proposed to reduce the computational cost of the method. Finally, the method is tested on several examples and found to be in good agreement with exact solution.

Contents

1. Introduction

In the last few years, there has been an increased interest in the application of Daubechies wavelets as bases to solve partial differential equations [1–6]. Daubechies scaling function with multiresolution analysis theory gives rise to Daubechies

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https://doi.org/10.1016/j.amc.2017.12.047
0096-3003/

wavelet. These wavelet bases have various properties such as orthogonality, compact support, arbitrary regularity and high order vanishing moments. These properties make wavelets natural choice in fields like signal processing, image processing, data compression, acoustics, seismology, nuclear engineering and biomedical engineering. In particular, these properties make wavelets efficient bases to solve differential and integral equations.

In 1990, Glowinski et al. [1] applied wavelet as a basis in variational setting to solve linear and nonlinear ordinary differential equations in one dimension. Amaratunga et al. [2] considered wavelet Galerkin technique to solve one dimensional partial differential equations with periodic and Dirichlet boundary conditions. Their investigations indicate that wavelet method is a strong competitor to the traditional methods like spectral method, finite element method, at least for those problems having simple domain. In 1993, Qian and Weiss [3,4] applied wavelet Galerkin method to solve elliptic partial differential equations in two dimension. Rathish and Mehra [5,6] applied this method to solve parabolic and hyperbolic partial differential equations with periodic boundary conditions.

In this paper, we propose a wavelet Galerkin method to solve higher order linear and nonlinear differential equations with periodic boundary conditions. These equations are generalization of biharmonic equation which appear in many engineering applications, such as the deformation of a thin plate and motion of a fluid. Nonlinear fourth order differential equations with periodic boundary conditions have many applications in dynamical systems. These periodic equations are theoretically investigated in [13,14] and references therein. Numerically these equations have been solved using Haar wavelet [11,12], but this wavelet is not smooth. To get the higher order accuracy, we need smooth functions as basis elements otherwise we will have to go to higher resolution level which increase the computational cost exponentially. To overcome this difficulty we apply Daubechies wavelets as bases to solve differential equations. Since we have exact value of 2-term connection coefficients, it is very easy to handle fourth order derivative when compared to other methods like finite element method, spectral method etc. Orthogonality and compact support of basis functions make the global matrix highly sparse. In particular, these properties of basis functions reduce the computational cost. Handling nonlinear term is always an important issue in nonlinear problems. In the nonlinear problem we obtain nonlinear wavelet coefficients which we have effectively calculated using quasilinearization technique. We, in fact, exploit the property of wavelet coefficients of linear equation in obtaining wavelet coefficients for nonlinear counterpart. To reduce the computational cost, we have proposed a compression algorithm.

The content of this paper is organized as follows. In Section 2, we give a brief introduction of Daubechies wavelets and 2-term connection coefficients. In Section 3, we describe wavelet Galerkin method for linear differential equations. In Section 4, we describe error and stability estimate. Various numerical examples with logarithmic scale error have been presented in Section 5. In Section 6, we propose wavelet Galerkin method for nonlinear differential equations. Numerical examples and logarithmic scale error have been presented. An efficient compression algorithm is proposed in this section.

2. Basic background

Multiresolution analysis is an important tool to construct an orthonormal basis of $L^2(\mathbb{R})$. Daubechies used this tool to construct a class of compactly supported wavelets with arbitrary regularity.

Definition 2.1 (see [9]). A multiresolution analysis (MRA) of $L^2(\mathbb{R})$ consists of a sequence of closed subspaces V_J $(J \in \mathbb{Z})$ of $L^2(\mathbb{R})$ satisfying

(i) $V_J \subset V_{J+1}$ for all $J \in \mathbb{Z}$;
(ii) $f \in V_J \Leftrightarrow f(2(\cdot)) \in V_{J+1}$ for all $J \in \mathbb{Z}$;
(iii) $\bigcap_{J\in\mathbb{Z}} V_J = \{0\}$;
(iv) $\overline{\bigcup_{J\in\mathbb{Z}} V_J} = L^2(\mathbb{R})$;
(v) There exists a function $\phi \in V_0$ such that $\{\phi(\cdot - k)\colon k \in \mathbb{Z}\}$ is an orthonormal basis for V_0.

The function ϕ is known as scaling function.
Approximation space V_J is defined as

$$V_J = \overline{\text{span}\{\phi_{J,k}(x) \in L^2(\mathbb{R}) \ \ | \ \ k \in \mathbb{Z}\}},$$

where

$$\phi_{J,k}(x) = 2^{J/2}\phi(2^J x - k), \qquad J, k \in \mathbb{Z}.$$

Wavelet space W_J is defined as the orthogonal complement of V_J in V_{J+1}, i.e.

$$V_J \perp W_J \text{ and } V_{J+1} = V_J + W_J.$$

Since $\phi(x) \in V_0 \subset V_1$ and $\psi(x) \in V_1$, we have two-scale relation

$$\phi(x) = \sqrt{2}\sum_{k=0}^{D-1} a_k \phi(2x - k),$$