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Dissipative fault-tolerant control for nonlinear singular perturbed systems with Markov jumping parameters based on slow state feedback☆

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ABSTRACT

This paper focuses on the analysis and design of dissipativity-based fault-tolerant controller for discrete-time nonlinear Markov jump singularly perturbed systems (MJSPSs) which are based on Takagi–Sugeno fuzzy model. A novel strategy is proposed to improve the upper bound of singular perturbation parameter (SPP) ϵ, and the fault-tolerant design is also introduced, namely the susceptible property of systems is made full consideration, to ensure the specified performance of a system. The aim is to design an optimized slow state feedback controller such that the stability of MJSPSs is guaranteed even in faulty case, and the upper bound of the SPP ϵ is improved simultaneously. Utilizing Lyapunov functional technique, a sufficient condition for the existence of controller is shown. Last but not least, the control issue of a series DC motor model as an illustrated example is given to explain the availability of the presented design scheme.

1. Introduction

In system theory and control engineering, provided that there are some small time constants, inertia, conductance, or capacitance in the system, the differential equations of a mathematical model are made to have a relatively high order and the ill-conditioned numerical characteristics. One of the tools to deal effectively with such problems is the singular perturbation method, whose standpoint is to firstly ignore the fast variables for reducing the order of the system. As a consequence, by introducing the coefficient of boundary layer correction, i.e., ϵ, which is exploited to determine the degree of separation between "slow" and "fast" modes of the system, the approximation is effectively improved. In consequence, the singularly perturbed systems (SPSs) have been studied extensively, see, [7,11,23,30,44].

However, a great number of products mentioned above are only applicable to linear SPSs but nonlinear SPSs deserve more attention in engineering. Obviously, it is imperative for nonlinear SPSs to express nonlinearity when analyzing dynamic behavior [17,29,30,42]. For this reason, the Takagi and Sugeno (T–S) fuzzy control systems [13,19,20,31,36], which are local

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linear time-varying systems connected by the IF-THEN rules, are recommended to describe nonlinear systems. The results in [9] had been presented that T–S fuzzy theory can be used to model a wide class of nonlinear systems with the advantage that the fuzzy systems can approach continuous function at any precision. The stability analysis and control design of T–S fuzzy SPSs were studied in literature [45]. Liu et al., Tuan and Hosoe [18,37] studied the $\mathcal{H}_\infty$ control of T–S fuzzy SPSs; [3] and [5] also considered the problem of control and pole configuration for T-S fuzzy SPSs simultaneously.

Note that, only when there is no random occurrence in the parameters of SPSs, the above results can be applied. Nevertheless, such a random occurrence phenomenon might be inevitable under many practical circumstance [6,27,32,34,46,47]. Fortunately, the Markov jump systems (MJSs) have the ability to model the systems, in which its structure or parameters mutate randomly [28,33]. As a result, taking it for granted that there exists a great number of investigations of MJSs [38]. Furthermore, taking the phenomenon into account in SPS, it is necessary to raise the ancient MJSs to Markov jump singular perturbation system (MJSPS). The $\mathcal{H}_\infty$ control problem for discrete-time MJSPSs was discussed by Wang and Zhang in [38], where the Markov approach was used to show the switching between diverse modes of controller, and the efficient conditions of stability and $\mathcal{H}_\infty$ performance of closed-loop system were also obtained. Combined with the foregoing, when the MJSPS is considered with the nonlinearity, the T–S fuzzy SPSs with Markov jumping parameters should be adopted, which is worth to be studied.

Owing to the complexity of the nonlinear MJSPSs, they are more susceptible leading to sensors, actuators or the controllers faults. Besides, in virtue of the closed-loop systems, the flaw mentioned above will also get worse, and faults could turn into an annular malfunction. When the traditional feedback controller encounters a fault, it is much easier to cause production shutdown or factory failure. Given this event, an effective program, fault-tolerant control (FTC) is taken into consideration to guarantee the system stability and safety [21]. The FTC not only guarantees perfectly stable performance of the system, but also overcomes the obstacle of traditional feedback control. However, the design of fault-tolerant controller for T–S fuzzy SPSs with Markov jumping parameters has not been fully considered, which is the main motivation of our paper.

Inspired by the above consideration, in this paper, by employing the slow state variables feedback scheme, we devote to investigating the dissipative control for discrete-time SPSs with Markov jump parameters. The contributions of this paper can be outlined as the following four points: (a) For making the established SPSs more general, the nonlinearity is also considered, which is modeled by the T–S fuzzy theory. (b) Furthermore, a new method is given to estimate and improve the upper bound of SPP ϵ, which gives a bigger stability bound. (c) In addition, regarding randomly occurring faults, one employs a stronger fault tolerance to the new design methods of controller. (d) Instead of considering single performance index, the extended dissipative performance index which contains different index cases as expounded in Remark 2 is introduced, bringing a wider adaptability for different indexes as discussed in Example 1.

The organization of this paper is given as follows. In Section II, the problems, which are taken into consideration in this paper, are described in detail. At the same time, the model of the nonlinear singularly perturbed Markov jump system based on T–S fuzzy theory is constructed as well. The analysis and design based on dissipative control for the discrete-time nonlinear MJSPS are conducted in Section III, in which the new methods are also obtained to evaluate the upper bound of SPP. Two numerical examples are provided in Section IV to illustrate the validity of our proposed approach. Finally, we conclude this paper in Section V.

Notation. The notations used throughout this paper: n_1 -dimensional Euclidean space is denoted by $\mathbf{R}^{n_1}$. The notation $Q_3>0$ indicates the matrix Q_3 is positive definite and real symmetric. Additionally, Q_3^T remarks the transpose of the matrix Q_3; I and 0 represent the identity and zero matrix with appropriate dimension, respectively; $\mathcal{E}\{\cdot\}$ means the mathematical expectation; the asterisk $(*)$ is employed to stand for a symbol which is caused by symmetry. Finally, matrices which are not explicitly stated are supposed to have appropriate dimensions.

2. Problem formulation

Firstly, the discrete-time SPSs are described by fuzzy system model (Σ_1):
Plant Rules i: IF $\theta_1(k)$ is V_{i1}, $\theta_2(k)$ is V_{i2},..., $\theta_o(k)$ is V_{io}, then

$$x_1(k+1)=A_i^{11}(\kappa(k))x_1(k)+\epsilon A_i^{12}(\kappa(k))x_2(k)+B_i^{\omega 1}(\kappa(k))\omega(k)+B_i^{u1}(\kappa(k))u(k), \tag{1}$$

$$x_2(k+1)=A_i^{21}(\kappa(k))x_1(k)+\epsilon A_i^{22}(\kappa(k))x_2(k)+B_i^{\omega 2}(\kappa(k))\omega(k)+B_i^{u2}(\kappa(k))u(k), \tag{2}$$

$$z(k)=C_i^1(\kappa(k))x_1(k)+\epsilon C_i^2(\kappa(k))x_2(k), \tag{3}$$

where $i=1,2,\dots,g$ and g denotes the number of IF-THEN rules of the system; the fuzzy sets are made up of $V_{ij}(j=1,2,\dots,o)$; and the premise variables are shown by $\theta_j(k)$ $(j=1,2,\dots,o)$; $x_1(k)\in\mathbf{R}^{n_1}$ represents the slow state vector and $x_2(k)\in\mathbf{R}^{n_2}$ is the fast one; $\omega(k)\in\mathbf{R}^u$ and $u(k)\in\mathbf{R}^v$ mean the disturbance and control input, respectively; $z(k)\in\mathbf{R}^p$ indicates the output; $\epsilon>0$, the singularly perturbation parameter (SPP), determines the degree of separation between the "slow" and "fast" modes of the SPSs. Additionally, with taking values in a finite set $\mathcal{S}=\{1,2,\dots,S\}$, $\kappa(k)$ means a discrete-time Markov chain in which transition probability matrix $\Pi\triangleq\{\pi_{rt}\}$ is given by

$$\pi_{rt}\triangleq\Pr\{\kappa(k+1)=t|\kappa(k)=r\}\geqslant 0,\qquad \forall t,r\in\mathcal{S},$$