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Global exponential synchronization of nonautonomous recurrent neural networks with time delays on time scales

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ABSTRACT

This paper is concerned on the global exponential synchronization in timescale sense for a class of nonautonomous recurrent neural networks (NRNNs) with discrete-time delays on time scales. Firstly, a timescale-type comparison result is given based on the induction principle of time scales. Then by the constructed comparison lemma, the theory of time scales and analytical techniques, several synchronization criteria for the driven and response NRNNs are obtained. Moreover, several examples are given to show the effectiveness and validity of the main results. The obtained synchronization criteria improve or extend some existing ones in the literature.

1. Introduction

In recent years, due to their wide applications in pattern classification, image processing, information science, and security communication, neural networks (NNs) have received many attentions in recent years [1–13]. Various kinds of dynamics have been investigated for NNs, such as boundedness [14,15], periodicity [14,16], stability [17–21], chaos [22], synchronization [23–27], and so forth. The concept of drive-response synchronization was firstly proposed for coupled chaotic systems in 1990 [28], and hereafter, the synchronization problem has been widely studied. Different approaches have been adopted to synchronize the drive-response systems, such as state feedback control [8], adaptive or fuzzy control [27], impulsive techniques [3,29], pinning control methods [24,30], to name just a few. In [17], the authors considered the stability for a class of continuous-time delayed NNs, in which a new Lyapunov-Krasovskii functional is constructed. In [23], Liu et al. derived a self synchronization criterion for a class of discrete-time delayed Hopfield NNs based on a newly constructed Halanay inequality. Among these literature, most of the neural networks are just discussed for the autonomous cases. However, it is believed that many models developed in mechanics, biology, and physics are formulated regarding nonautonomous dynamical systems. For example, when considering the long-term dynamic behaviors, the system parameters usually change along with time [31], rather than keep unchanged all the time. Therefore, for neural network models, the nonautonomous recurrent neural networks are necessary to be discussed.

As we know, continuous-time and discrete-time NNs are often separately investigated as there are some intrinsic differences between differential equations (continuous-time NNs) and difference equations (discrete-time NNs). However, there

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exist plenty of systems that contain continuous-time and discrete-time domains on the time evolution. For instance, some neurons in our brain are active in the day, while the will be inactive at night and they will be anew in the next day, cycling like that [32,33]. Furthermore, for a continuous-time system, its property may be deduced by mathematics, but when implemented in computers, it will be transformed into an approximate discrete-time system as all the information in computers is proceeded in the form of discrete-time signals. Therefore, it is necessary and vital to consider a system that contains both continuous-time and discrete-time systems. To this end, a critical tool, known as time scale, was introduced in [34]. The time scale framework primarily aimed to unify the differential and difference equations. In fact, investigations of systems on time scales not only cover the continuous-time and discrete-time results but also hold the results for the systems that involve on time-interval domain. In recent years, many results have been obtained based on the time scales [32,33,35–44]. The authors in [36] considered the multiperiodicity and activating set for a class of linear threshold networks which covered the continuous-time results in [45] and the discrete-time result in [46]. We can see that investigations of NNs on time scales can eliminate many repetitious works. As far as the authors' knowledge, some works have been done about the global exponential synchronization for autonomous recurrent neural networks [26,37,47,48], but few works have been published about the global exponential synchronization for nonautonomous recurrent neural networks on time scales. To enrich and complement some previous results on autonomous neural networks and to integrate the results for continuous-time and discrete-time nonautonomous neural networks are two major motivations of this paper.

Based on the statements mentioned above, this paper will investigate the global exponential synchronization in timescale sense of NRNNs with discrete delay. Roughly stated, the main contributions of this paper can be summarized as the following threefold. (i) A timescale type nonautonomous comparison principle is developed based on the induction of time scales, and it plays a key role in our main results. (ii) By constructing Lyapunov function and using the theory of time scales, a synchronization criterion in timescale sense is obtained for NRNNs. (iii) By the classification of a point on time scales, another timescale-type synchronization criterion is obtained, in which Lyapunov function is not necessary to construct. Moreover, they not only hold for continuous-time NRNNs and discrete-time NRNNs, but also hold for the systems that involved on the time-interval domain.

The organization of this paper is as follows. In Section 2, some preliminaries are recalled, and the problem is formulated. In Section 3, the global exponential synchronization in timescale sense is analyzed for NRNNs with discrete delays. Four examples are given to verify the effectiveness of the results in Section 4, and followed by conclusions in Section 5.

2. Basic concepts and problem formulation

Firstly, we introduce some notations: $\mathbb{R}$ and $\mathbb{Z}$ are the real number set and the integer set, resp. $\mathcal{I}_n := \{1, 2, \dots, n\}$. $\bar{y}(t) = \max_{i,j}\{y_{ij}(t)\}$ for any $t \in \mathbb{T}$. $\tilde{k} = \max\{k_1, k_2, \dots, k_n\}$.

2.1. Basic concepts

Some basic concepts of time scales are recalled which are essential for the main results. For more details, one can refer to [32,33].

A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of the real set $\mathbb{R}$. The forward and backward jump operators $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ are defined by $\sigma(t) = \inf\{w \in \mathbb{T} : w > t\}$ and $\rho(t) = \sup\{w \in \mathbb{T} : w < t\}$, respectively. The graininess function $\mu : \mathbb{T} \to [0, +\infty)$ is defined by $\mu(t) := \sigma(t) - t$.

A point t in $\mathbb{T}$ is said to be right- (or left-) dense if $\sigma(t) = t$ (or $\rho(t) = t$); A point t in $\mathbb{T}$ is said to be right- (or left-) scattered if $\sigma(t) > t$ (or $\rho(t) < t$). If $\mathbb{T}$ has a left-scattered maximum T then $\mathbb{T}^k := \mathbb{T}/\{T\}$, otherwise $\mathbb{T}^k := \mathbb{T}$. A function $f : \mathbb{T} \to \mathbb{R}$ is called rd-continuous if it is continuous at right-dense points and the left-hand sided limits exist at left-dense points. The set of rd-continuous functions $f : \mathbb{T} \to \mathbb{R}$ is denoted by $C_{rd} = C_{rd}(\mathbb{T}, \mathbb{R})$. A function $p : \mathbb{T} \to \mathbb{R}$ is regressive provided $1 + \mu(t)p(t) \neq 0$ for all $t \in \mathbb{T}^k$ holds. The set of all regressive and rd-continuous functions on $\mathbb{T}$ is denoted by $\mathcal{R} = \mathcal{R}(\mathbb{T}, \mathbb{R})$. Moreover, $p \in \mathcal{R}^+$ if $1 + \mu(t)p(t) > 0$ for all $t \in \mathbb{T}^k$. For any interval $[a, b]$ on $\mathbb{R}$, $[a, b]_{\mathbb{T}}$ means the intersection of the interval $[a, b]$ and the time scale $\mathbb{T}$, i.e., $[a, b]_{\mathbb{T}} = [a, b] \cap \mathbb{T}$.

Definition 1. [32] Suppose $f : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{T}^k$. If there exists a number $f^{\Delta}(t)$ admitting that, for any $\varepsilon > 0$, there exists a neighborhood U of t so that

$$|f^{\sigma}(t) - f(\xi) - f^{\Delta}(t)(\sigma(t) - \xi)| \le \varepsilon|\sigma(t) - \xi|, \ \forall \xi \in U,$$

is said to be the $\Delta-$derivative of f at t, where $f^{\sigma}(t) = f(\sigma(t))$. And in such case, we say that f is Δ-differentiable at t. If the neighborhood U is replaced by the right-hand sided neighborhood U^+, then the derivative is said to be the Dini derivative, and denoted by $D^+_{\Delta} f(t)$.

Lemma 1. *[32] Assume $f, g : \mathbb{T}^k \to \mathbb{R}$ are differentiable at $t \in \mathbb{T}^k$. Then*

(i) The sum $f \pm g : \mathbb{T}^k \to \mathbb{R}$ is differentiable at t with

$$(f \pm g)^{\Delta}(t) = f^{\Delta}(t) \pm g^{\Delta}(t).$$

(ii) For any constant α, $\alpha f : \mathbb{T}^k \to \mathbb{R}$ is differentiable at t with

$$(\alpha f)^{\Delta}(t) = \alpha f^{\Delta}(t).$$