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Tripoly Stackelberg game model: One leader versus two followers

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ABSTRACT

This paper is devoted to introduce and study a Stackelberg game consisting of three competed firms. The three firms are classified as a leader which is the first firm and the other two firms are called the followers. A linear inverse demand function is used. In addition a quadratic cost based on an actual and announced quantities is adopted. Based on bounded rationality, a three-dimensional discrete dynamical system is constructed. For the system, the backward induction is used to solve the system and to get Nash equilibrium. The obtained results are shown that Nash equilibrium is unique and its stability is affected by the system's parameters by which the system behaves chaotically due to bifurcation and chaos appeared. Some numerical experiments are performed to portrays such chaotic behavior. A control scheme is used to return the system back to its stability state and is supported by some simulations.

1. Introduction

Dynamic games carried out in economic market are important because of the complex dynamic characteristics appeared in such games. Two factors in those games should be highlighted, the behaviors on which consumers use and the reactions performed by the competitors. Recent studies have investigated the dynamic behavior of those games such as monopolistic [1], duopolistic [2–9], and tripolistic one [10,11]. Literature has shown that only few studies are dealing with the tripolistic models. The current paper introduces and studies a tripolistic model based on Stackelberg assumptions.

In 1934, the German economist Heinrich Freiherr von Stackelberg introduced the so-called Market Structure and Equilibrium that in turns described the Stackelberg model. Stackelberg game is a strategic game in economics in which one firm of the competed firms is called the leader and moves first and then the other firms which they are called the follower firms move sequentially. In such games, it must be a firm that has some sort of advantage making it leads the market and takes the first move. In other words, the leader must have power of commitment. To be the firm that has the first move you should be the incumbent monopolistic firm of the commodity and your competitors (followers) are new entrants. In repeated Stackelberg game if the follower feels that he/she might be punished he/she may adopt a punishment strategy as well to hurt the leader unless the leader chooses a non-optimal strategy in the current period. One has to mention here that both Cournot and Stackelberg games are similar as both use the same decisional variables that are the quantities. However, in the Stackelberg, there is a crucial advantage for the leader as he/she starts the first move in the market. Furthermore,

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the perfect information is also important in such game because the follower must watch the quantity chosen by the leader otherwise Stackelberg game will be reduced to Cournot.

In formulating economic models one should be aware that expectation is an important role while describing such models. There are several techniques that may be used by the firms to adjust and update their productions. For instance, Puu's strategy [12] and bounded rationality mechanism [13]. The current paper adopts the bounded rationality approach to introduce a triopoly Stackelberg game and then investigate the complex dynamic characteristics of the game. The proposed game consists of three firms that are competed. The first firm is chosen to lead the game (the leader) and the other two firms (the followers) follow the first move carried out by the leader. The three firms update their productions so as to maximize their profits in the subsequent stages. Depending on the expected marginal profits the firms adjust their decisional variables and of course they use local information about their outputs. In other words, the stackelberg firms maximize their profits based on the local information of their decisional variables.

Recent trends in tackling economic competition have adopted networks of networks or multilayer networks. They are aptly used to describe such competition or social systems. In literature, multilayer networks have been devoted to evolutionary games and in particular to the evolution of cooperation. For instance, in [29] a colloquium has been introduced to highlight some important aspects of cooperation under adverse conditions, as well as the cooperative interaction among groups in evolutionary game theory. In [30], a concise and informative review on coevolutionary games has discussed. Other interesting articles are available in literature [31–34].

Outline of the current paper is as follows. In Section 2, a triopoly game is introduced based on Stackelberg assumptions and bounded rationality mechanism. All the equilibrium positions of the system described the game are obtained. Analysis of the local stability of the system's equilibrium points are illustrated. Some numerical simulations to confirm and verify the obtained theoretical results are performed in Section 2. In Sections 3, we apply control scheme on the proposed system to suppress the chaotic behavior appeared in the system. Finally, some concluding remarks are shown.

2. Model

We suppose that there are three firms labelled by i, $i = 1, 2, 3$. Those firms produce the same commodities so as to sale them in the market. Assuming that firm 1 leads the competition (Stackelberg leader) among the firms and firm 2 and firm 3 are two followers. Decisional variables of the three firms are production quantities and are denoted by $q_{i,t}$, $i = 1, 2, 3$ which are updated according to discrete time steps, $t \in \mathbb{Z}_+$. The price is determined by the total supply $Q = \sum_{i=1}^{3} q_{i,t}$ at time period t, and is given by,

$$p_t = a - b\sum_{i=1}^{3} q_{i,t} \tag{1}$$

where, a and b are positive constants. Indeed, $q_{i,t}$ should be positive because negative quantities do not have any valuable meaning in economy and therefore positive price is guaranteed when $Q < \frac{a}{b}$. From an economic prospective, the announced products are different than the actual products for firm i at time t. For the leader the announced products may be less than the actual products but for the followers the converse may be true. Using different strategy of productions, we assume that the firms use the following cost function.

$$C_i(q_i) = c_i(q_{i,t} - \theta_i)^2, i = 1, 2, 3 \tag{2}$$

and c_i, $i = 1, 2, 3$ is a positive parameter. As in [14], θ_i, $i = 1, 2, 3$ refers to the announced plan products of firm i, $i = 1, 2, 3$ respectively. Now, the profit of each firm can be written as,

$$\pi_{i,t}(q_1, q_2, q_3) = q_{i,t}\left[a - b\sum_{i=1}^{3} q_{i,t}\right] - c_i(q_{i,t} - \theta_i)^2, i = 1, 2, 3 \tag{3}$$

The target of each firm is to maximize its profit and this requires to estimate the marginal profit as follows,

$$\frac{\partial \pi_{i,t}}{\partial q_i} = a + 2c_i\theta_i - 2(b + c_i)q_i - b\sum_{j\neq i}^{3} q_i, i = 1, 2, 3; i \neq j \tag{4}$$

So, to maximize the profit we solve the system of algebraic equations, $\frac{\partial \pi_{i,t}}{\partial q_i} = 0, i = 1, 2, 3$ then we get the following,

$$\begin{aligned}
q_{1,t} &= \frac{2c_1\theta_1[b(3b + 4c_2) + 4c_3(b + c_2)] - 2b[c_2\theta_2(b + 2c_3) + c_3\theta_3(b + 2c_2)] + a(b + 2c_2)(b + 2c_3)}{2[b^2[2b + 3(c_1 + c_2 + c_3)] + 4bc_3(c_1 + c_2) + 4c_1c_2(b + c_3)]},\\
q_{2,t} &= \frac{2c_2\theta_2[b(3b + 4c_1) + 4c_3(b + c_1)] - 2b[c_1\theta_1(b + 2c_3) + c_3\theta_3(b + 2c_1)] + a(b + 2c_1)(b + 2c_3)}{2[b^2[2b + 3(c_1 + c_2 + c_3)] + 4bc_3(c_1 + c_2) + 4c_1c_2(b + c_3)]},\\
q_{3,t} &= \frac{2c_3\theta_3[b(3b + 4c_1) + 4c_2(b + c_1)] - 2b[c_1\theta_1(b + 2c_2) + c_2\theta_2(b + 2c_1)] + a(b + 2c_1)(b + 2c_2)}{2[b^2[2b + 3(c_1 + c_2 + c_3)] + 4bc_3(c_1 + c_2) + 4c_1c_2(b + c_3)]},
\end{aligned} \tag{5}$$