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Time-fractional generalized Boussinesq equation for Rossby solitary waves with dissipation effect in stratified fluid and conservation laws as well as exact solutions ☆

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ABSTRACT

Construct fractional order model to describe Rossby solitary waves can provide more pronounced effects and deeper insight for comprehending generalization and evolution of Rossby solitary waves in stratified fluid. In the paper, from the quasi-geostrophic vorticity equation with dissipation effect and complete Coriolis force, based on the multi-scale analysis and perturbation method, a classical generalized Boussinesq equation is derived to describe the Rossby solitary waves in stratified fluid. Further, by employing the reduction perturbation method, the semi-inverse method, the Agrawal method, we derive the Euler–lagrangian equation of classical generalized Boussinesq equation and obtain the time-fractional generalized Boussinesq equation. Without dissipation effect, by using Lie group analysis method, the conservation laws of time-fractional Boussinesq equation are given. Finally, with the help of the improved (G'/G) expansion method, the exact solutions of the above equation are generated. Meanwhile, in order to consider the dissipation effect, we have to derive the approximate solutions by adopting the New Iterative Method. We remark that the fractional order model can open up a new window for better understanding the waves in fluid.

1. Introduction

Nowdays, Rossby solitary waves are of high concern to meteorologists and oceanographers to a large extent. Rossby solitary waves are very important phenomenon in the ocean and the atmosphere, determining the ocean's response to the atmosphere and climate change. Rossby solitary waves in the atmosphere and marine science have important theoretical and practical significance, for fishing, aquatic and the reallocation of marine energy. In recent years, Rossby solitary waves have evolved from the theoretical description of evolution, describing the classical Rossby solitary waves with the Boussinesq equation, KDV equation, mKDV equation [1–3]. The expressions of algebraic solitary waves pass through the BO equation and

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the ILW equation [4–6]. However, these pioneers are more research on the Rossby solitary waves integer order model. In order to reconcile with the actual ocean process, it's necessary to introduce the ability of a fractional differentiation. As we know, fractional calculus is local, representing non-conservative forces, one of the general characteristics of the classical calculus. Fractional calculus is applied in almost all areas of mathematics, science and engineering, including biology, electrical engineering, viscoelasticity and rheology signal and image processing, control theory, etc [7–12]. The reason why nonlinear partial differential equations are increasingly concerned is that they can accurately describe non-linear phenomena [13–20]. In particular, not only depend on the time but also the various historical phenomena relate to real-time problems in previous time history, they can pass the time-fractional nonlinear partial differential equations successfully modeled. At present, there are limited studies on the fractional model of Rossby solitary waves in the ocean atmosphere. Our aim is to study the propagation of Rossby solitary waves using the time-fractional generalized Boussinesq equation with dissipation effect from the quasi-geostrophic vorticity equation in stratified fluid under the influence of complete Coriolis force and dissipation, and to compare the influence of the time fractional parameters. This work has important theoretical and practical value.

The conservation laws are developed from the physical principle of conservation laws are also used in the development of numerical methods to establish the uniqueness and existence of solutions. There are many ways to study the PDE conservation laws, such as local lagrangian method [21,22], multiplier method [23], Noether method [24–27], and there are also many methods to solve the fractional differential equation, like the first integral method [28], the function variable method [29], the sub-equation method [30], the trial function method [31] and so on [32–38].

The structure of the paper is as follows: the model of integer order is obtained by perturbation reduction method in Section 2. Using the semi-inverse method, the Agrawal method and the Euler–Lagrangian equation [39,40] to get the fractional order model in Section 3. Without considering the dissipation effect, by using Lie group analysis method [41,42], the conservation laws of time-fractional generalized Boussinesq equation are given in Section 4. Using the improved (G'/G) expansion method [43] to obtain the exact solutions of the equation, consider using the New Iterative Method [44] to get the approximate solution under dissipation effect in Section 5. Finally some conclusions will be placed in Section 6.

2. Dissipative Boussinesq equation

2.1. Mathematical modeling and dimensionless

Starting from the quasi-geostrophic vorticity equation with dissipation effect and complete Coriolis force in stratified fluid, it can be written as [45]

$$\left(\frac{\partial}{\partial t}+\frac{\partial\Psi}{\partial x}\frac{\partial}{\partial y}-\frac{\partial\Psi}{\partial y}\frac{\partial}{\partial x}\right)\left[\nabla^2\Psi+\frac{f^2}{\rho_s}\frac{\partial}{\partial z}\left(\frac{\rho_s}{N^2}\frac{\partial\Psi}{\partial z}\right)+\beta(y)y-f_H\frac{\partial B}{\partial y}\right]=0. \tag{1}$$

Where Ψ is the stream function; f is the Coriolis parameter; $N^2(z)=-\frac{g}{\rho_s}\frac{\partial\rho_s}{\partial z}$ is the Brunt–Vaisala frequency, which represents a measure of stability of the stratification; ρ_s indicates density; $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ is the two-dimensional laplace operator; $\beta(y)y$ and f_H are the vertical component and the horizontal component of the Coriolis parameter, respectively, and f_H is constant; $B(x, y)$ denotes the bottom topography.

Introducing the dimensionless as

$$(x,y)=L(x^*,y^*),\quad \Psi=LU_0\Psi^*,\quad t=\frac{L}{U_0}t^*,\quad \beta=\frac{U_0}{L^2}\beta^*,\quad B=\frac{U_0H}{f_HL}B^*, \tag{2}$$

where L is the zonal characteristic length, H is the vertical characteristic length, and U_0 is the characteristic velocity. In addition, the variables with asterisk are dimensionless. Assume that the aspect ratio is $\delta=\frac{H}{L}$ and substituting Eq. (2) into Eq. (1) gets

$$\left(\frac{\partial}{\partial t}+\frac{\partial\Psi}{\partial x}\frac{\partial}{\partial y}-\frac{\partial\Psi}{\partial y}\frac{\partial}{\partial x}\right)\left[\nabla^2\Psi+\frac{f^2}{\rho_s}\frac{\partial}{\partial z}\left(\frac{\rho_s}{N^2}\frac{\partial\Psi}{\partial z}\right)+\beta(y)y-\delta\frac{\partial B}{\partial y}\right]=0. \tag{3}$$

Here for the convenience of writing, we remove the dimensionless asterisk.

The lateral boundary condition satisfies

$$\frac{\partial\Psi}{\partial x}=0,\quad y=0,L_1. \tag{4}$$

The upper boundary condition in the vertical direction is

$$\rho_s\Psi\to 0,\quad z\to\infty. \tag{5}$$

The lower boundary condition can be obtained by the thermal equation

$$\left(\frac{\partial}{\partial t}+\frac{\partial\Psi}{\partial x}\frac{\partial}{\partial y}-\frac{\partial\Psi}{\partial y}\frac{\partial}{\partial x}\right)\frac{\partial\Psi}{\partial z}+\mu_0\frac{N^2}{f}\nabla^2\Psi=q',\quad z=0. \tag{6}$$

Where $\mu_0\frac{N^2}{f}\nabla^2\Psi$ is defined as dissipation effect, $\mu_0\geq 0$ represents the dissipation factor, and q' is used to eliminate the dissipation of the elementary stream function and $q'=-\mu_0\frac{N^2}{f}\frac{\partial U_1}{\partial y}$.