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Multiscale analysis and computation for coupled conduction, convection and radiation heat transfer problem in porous materials☆

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ABSTRACT

This paper discusses the multiscale analysis and numerical algorithms for coupled conduction, convection and radiation heat transfer problem in periodic porous materials. First, the multiscale asymptotic expansion of the solution for the coupled problem is presented, and high-order correctors are constructed. Then, error estimates and their proofs will be given on some regularity hypothesis. Finally, the corresponding finite element algorithms based on multiscale method are introduced and some numerical results are given in detail. The numerical tests demonstrate that the developed method is feasible and valid for predicting the heat transfer performance of periodic porous materials, and support the approximate convergence results proposed in this paper.

1. Introduction

With the rapid advance of material science and technology, porous materials are of importance in engineering and industry owing to their high temperature resistance, low thermal conductivity and light weight etc. Especially, with rapid development of space aircraft, porous materials designed as insulation for thermal protection system (TPS) have attracted tremendous attention and wide research interest in practical engineering applications. Therefore, it is essential to accurately predict the physical performance of the porous materials.

The ways of heat transfer in porous materials contain conduction, convection and radiation. For porous materials at low temperature, heat radiation can be neglected for most practical purposes. However, at higher temperatures when the solid phase is poor heat conductor and the pores are large, an accurate calculation should consider the heat transfer by radiation. Considering the interior surface radiation of porous materials, some numerical and theoretical methods were introduced in the past years [1,2]. Convective heat transfer is generated by flow of fluids and can be ignored at low pressure or in closed-cell porous materials [1,3]. But convection is significant in heat transfer as the pores in materials are large or connected [4]. El Ganaoui [5] studied a linear heat conduction equation with convection boundary condition using homogenization method, however did not show the convergence and performed the first-order asymptotic expansions. Terada et al. [6] considered

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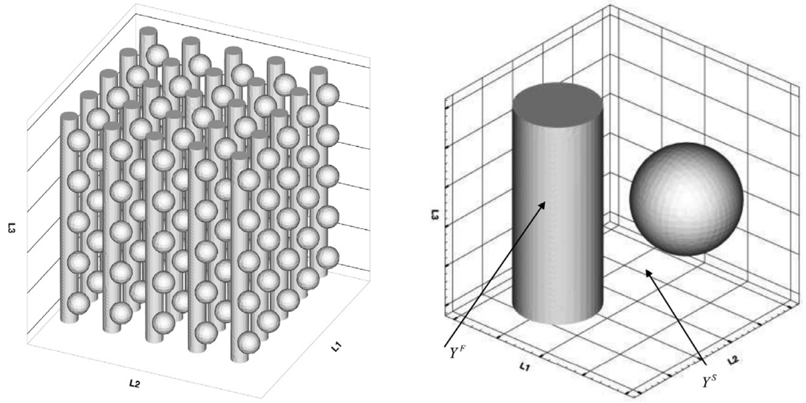

(a) (b)

Fig. 1. Periodic distribution of porous materials.

the scale effect of unit cells and derived formal expansions for thermo-mechanical coupling problem with small parameter ε. Su et al. [7] proposed a multiscale asymptotic expansion for the coupled conduction and convection problem with rapidly oscillatory coefficients, and derived the convergence results with an explicit rate $\varepsilon^{1/2}$.

To solve the coupled conduction, convection and radiation heat transfer problem of porous materials by classical numerical computation becomes rather difficult because it requires fine meshes and prohibitive amount of computation time. We know that the basic idea of the homogenization method is to give the overall behavior of the porous materials by incorporating the fluctuations due to the heterogeneities, which can not only save the computational resources but also ensure the calculation accuracy. Based on traditional homogenization method [8,9], Allaire and Habibi [10] discussed a heat transfer problem with non-linear radiation boundary conditions by two-scale asymptotic expansion, and gave the homogenized solutions and first-order correctors, but high-order correctors are not presented. Cui et al. [11,12] presented a high-order multiscale analysis method to predict the physical and mechanical properties of composite materials with periodic configuration, and solved some practical engineering problems. By high-order correctors, the microscopic fluctuation of physical and mechanical behavior inside the material can be captured more accurately.

It should be pointed out that homogenization method only describes the asymptotic behavior of the problems as $\varepsilon \to 0$. However, in practical engineering computation, ε does not approach to zero, and in fact is not very small. Numerous numerical results (see, e.g., [11–13]) have shown that the numerical accuracy of the standard homogenization method may not be satisfactory if ε is not sufficiently small. In addition, if substituting the first-order multiscale solution into original equation, one can find that the residual is $O(1)$ even though H^1-norm of its error is $O(\varepsilon^{1/2})$. The local error $O(1)$ is not acceptable for engineers who want to capture the local behavior of the solution. Therefore, it is necessary to seek high-order asymptotic methods.

In this paper, we mainly discuss the coupled conduction, convection and radiation heat transfer problem in a three-dimensional (3-D) domain, periodically perforated by thin parallel pipes in which fluid is flowing. Heat convection occurs in the parallel thin pipes, and radiation is modeled by a non-linear boundary condition on the pipes' walls and the surfaces of cavities. The goal of this paper is to establish a novel multiscale method to give a better approximation. We introduce the high-order correction terms into the asymptotic expansion of the temperature fields, and obtain the approximate order of $O(\varepsilon^{1/2})$ on some regularity hypothesis. It should be pointed that the error estimates in H^1-norm is $O(\varepsilon^{1/2})$ due to its boundary error.

The remainder of this paper is outlined as follows. Section 2 is devoted to the formulations of the multiscale asymptotic expansion. In Section 3 the mathematical proof of the error estimates for the multiscale approximate solution is derived. Finally, the algorithm procedure and the numerical results for the coupled conduction, convection and radiation heat transfer are shown in Section 4.

Throughout the paper the Einstein summation convention on repeated indices is adopted. By C we shall denote a positive constant independent of ε.

2. Multiscale asymptotic expansion

2.1. Asymptotic expansion for the temperature fields

The domain Ω_ε has pipes with axis in the third direction, the diameter of which is of the same order as the period. The reference periodicity cell Y^* is decomposed in two parts: the holes Y^F occupied by a fluid (see Fig. 1(b)) and the porous solid matrix Y^S. We denote by Γ^1 the boundary between Y^S and Y^F. Then, we define the fluid domain Ω_ε^F as the pipe's domain