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$\mathcal{H}_\infty$ consensus for nonlinear stochastic multi-agent systems with time delay

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ABSTRACT

This paper is concerned with the problem of $\mathcal{H}_\infty$ consensus for nonlinear stochastic multi-agent systems with time-delay. The objective is to design a dynamic output feedback protocol such that the multi-agent system reaches consensus in mean square and has a prescribed $\mathcal{H}_\infty$ performance level. First, by transforming models, the $\mathcal{H}_\infty$ consensus problem is converted to a standard $\mathcal{H}_\infty$ control problem. Then, by using the Lyapunov–Krasovskii functional method and the generalized Itô's formula, both delay-independent and delay-dependent stochastic bounded real lemmas are developed. Based on these, sufficient conditions on the existence of the desired dynamic output feedback protocol are presented in the form of linear matrix inequalities. Finally, two numerical examples are given to illustrate the effectiveness of the proposed results.

1. Introduction

Cooperative control of multi-agent systems has attracted lots of attention from the systems and control community over the past few decades. This is due to its wide applications in diverse areas such as formation control, optimization, sensor networks, and cooperative surveillance [1–3]. In order to accomplish complex tasks, which cannot be done by a single agent, dynamic agents of a networked multi-agent system should communicate and coordinate with each other. Consensus of multi-agent systems, which means that all agents in a multi-agent system eventually reach an agreement on certain values of interest, plays a significant role in the cooperative control. Therefore, many efforts have been made in the research of consensus for multi-agent systems in recent years. To mention a few, distributed average consensus via gossip algorithm with real-valued and quantized data was analyzed in [4]; finite-time consensus for multi-agent systems with double-integrator dynamics was considered in [5]; leader-following consensus via sampled-data control with randomly missing data was studied in [6]; couple-group consensus for discrete-time multi-agent systems was investigated in [7], where both the fixed topology case and the stochastic switching topology case were discussed.

On the other hand, the last 20 years have witnessed significant advances in the control theory of stochastic systems. $\mathcal{H}_\infty$ control for stochastic systems via state feedback was studied in [8], where both infinite and finite horizon control design methods were presented; robust resilient $\mathcal{L}_2 - \mathcal{L}_\infty$ control for uncertain stochastic systems with multiple time delays was addressed in [9], where output feedback control design approaches were proposed; fault-tolerant control for stochastic systems subjected both to white noise disturbance and Markovian jumping was considered in [10], where an observer-based

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mode-dependent control scheme was developed. In the context of stochastic multi-agent systems, average-consensus conditions were proposed in [11] by using probability limit theory and algebraic graph theory; the asymptotic unbiased mean-square average consensus was considered in [12], where a stochastic approximation-type consensus protocol was utilized to attenuate noises; a necessary and sufficient condition for the stochastic consensus stabilization was established in [13]; the event-triggered consensus control was investigated in [14], where both the output feedback controller and the threshold of the triggering condition were designed such that the multi-agent system achieves consensus in probability.

It is noted that the authors in [11–14] did not consider the effects of time delays and nonlinearities. However, time delays are always encountered in a control system and may result in instability and poor system performance [15–19]. In addition, most dynamic systems are inherently nonlinear in nature [20–22]. As pointed out in [23], contrast with the linear systems, nonlinear systems may appear chaotic, counterintuitive, or unpredictable. To the authors' knowledge, there is little result on the consensus of stochastic multi-agent systems with time delays and nonlinearities, which remains important and challenging. Motivated by the observation, we are concerned with the $\mathcal{H}_\infty$ consensus problem of a class of nonlinear stochastic time-delay multi-agent systems in this paper. Specifically, we focus on the problem of the design of a dynamic output feedback (DOF) protocol such that the multi-agent system reaches consensus in mean square and has a prescribed $\mathcal{H}_\infty$ performance level. First, by transforming models, the $\mathcal{H}_\infty$ consensus problem is converted to a standard $\mathcal{H}_\infty$ control problem. Then, by using the Lyapunov–Krasovskii functional method and the generalized Itô's formula, both delay-independent and delay-dependent stochastic bounded real lemmas are developed. On the basis of these results, sufficient conditions for the solvability of the $\mathcal{H}_\infty$ consensus problem are proposed. It is worth mentioning that the desired gain matrix can be constructed through the numerical solution of a set of linear matrix inequalities (LMIs). In addition, the proposed design methods do not impose any extra constraints on system parameters. Finally, the effectiveness of the proposed results is illustrated with two numerical examples.

Notation. Let $\mathcal{G}=(\mathcal{V},\ \mathcal{E})$ be a directed graph of order n, where $\mathcal{V}=\{v_1,\ \ldots,\ v_n\}$ and $\mathcal{E}\subseteq\{(v_j,\ v_i):\ v_j,\ v_i\in\mathcal{V},\ v_j\neq v_i\}$ denote, respectively, the set of nodes and the set of edges. The adjacency matrix $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{n\times n}$ is defined such that $a_{ij}>0$ if $(v_j,\ v_i)\in\mathcal{E}$ and $a_{ij}=0$ otherwise; the Laplacian matrix is defined as $L=[l_{ij}]\in\mathbb{R}^{n\times n}$, where $l_{ii}=\sum_{j=1,\ j\neq i}^{n}a_{ij}$ and $l_{ij}=-a_{ij}$ for all $i\neq j$. For the edge $(v_j,\ v_i)$, node v_j is a parent of node v_i. Denote $\mathcal{N}_i=\{v_j:\ v_j\in\mathcal{V},\ (v_j,\ v_i)\in\mathcal{E}\}$ as the set of all parents of node v_i. Graph $\mathcal{G}$ is said to be balanced if $\sum_{j=1}^{n}a_{ij}=\sum_{j=1}^{n}a_{ji}$ for all $i=1,\ \ldots,\ n$. A directed path in $\mathcal{G}$ is a sequence of edges of the form $(v_{i_1},\ v_{i_2})$, $(v_{i_2},\ v_{i_3})$, $\cdots$. A directed tree is a directed graph in which every node has exactly one parent except for one node, called the root, which has no parent and which has a directed path to every other node. A directed spanning tree of $\mathcal{G}$ is a directed tree that contains all nodes of $\mathcal{G}$.

Let $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq 0})$ be a complete probability space with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$, $\mathfrak{E}\{\cdot\}$ the expectation operator, $||\cdot||$ the Euclidean norm of a vector and its induced norm of a matrix, $\mathcal{L}_2[0,\ \infty)$ the space of square-integrable vector functions over $[0,\infty)$ and its norm is denoted by $|\cdot|_2$. For a matrix X, we denote its transpose by X^T , its inverse (when it exists) by X^{-1}, and its smallest and largest eigenvalues by $\lambda_m(X)$ and $\lambda_M(X)$, respectively. If X is symmetric, then $X\geq 0$ (respectively, $X>0$) means that the matrix X is positive semi-definite (respectively, positive definite); if X is a square matrix, then $He(X)$ is defined as $He(X)=X+X^T$. Denote by 1_n the $n\times 1$ column vector of all ones, by 0_n the $n\times 1$ column vector of all zeros, by I_n the n-dimensional identity matrix, by $0_{n\times m}$ the $n\times m$-dimensional zero matrix, by $L_c=\left[\phi_{ij}\right]\in\mathbb{R}^{n\times n}$ *the symmetric matrix with* $\phi_{ii}=(n-1)/n$ *and* $\phi_{ij}=-1/n$ *for all* $i\neq j$, and by $diag\{X_1,\cdots,X_n\}$the block-diagonal matrix with $X_1,\ \cdots,\ X_n$ on its diagonal. In symmetric block matrices or long matrix expressions, we use an asterisk * to represent a term that is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. Preliminaries

2.1. Problem description

Consider a multi-agent system consisting of n agents, each with dynamic model:

$$\begin{aligned} dx_i(t)&=\big(Ax_i(t)+A_\tau x_i(t-\tau)+\delta(x_i(t))+Bu_i(t)+Dv_i(t)\big)dt+\epsilon(x_i(t))d\omega(t),\\ y_i(t)&=C_1x_i(t),\\ z_i(t)&=C_2\left(x_i(t)-\frac{1}{n}\sum_{j=1}^{n}x_j(t)\right),i=1,\cdots,n, \end{aligned}\tag{1}$$

where A, A_τ, B, C_1, C_2 and D are constant matrices with compatible dimensions; $x_i(t)\in\mathbb{R}^m$ is the state; $u_i(t)\in\mathbb{R}^p$ is the control input or protocol; $v_i(t)\in\mathbb{R}^q$ is the disturbance input which belongs to $\mathcal{L}_2[0,\infty)$ [24,25]; $y_i(t)\in\mathbb{R}^l$ is the output; $z_i(t)\in\mathbb{R}^r$ is the controlled output; $\omega(t)$ is an Brownian motion defined on $(\Omega,\ \mathcal{F},\ P)$; function $\delta(x_i(t))$ is assumed to satisfy:

$$\|\delta(a)-\delta(b)\|\leq\beta\|a-b\|,\tag{2}$$

for arbitrary $\forall a,b\in\mathbb{R}^m$, β is a positive constant. To guarantee the existence and uniqueness of the solution, function $\epsilon(x_i(t))$ is required to satisfy both the Lipschitz condition and the linear growth condition [26]:

$$\|\epsilon(a)-\epsilon(b)\|\leq\alpha\|a-b\|,\tag{3}$$