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Numerical simulation for coupled systems of nonlinear fractional order integro-differential equations via wavelets method

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ABSTRACT

In this paper, a new method for solving coupled systems of nonlinear fractional order integro-differential equations is proposed. The idea is to use Bernoulli wavelets and operational matrix. The main purpose of the technique is to transform the studied systems of fractional order integro-differential equations into systems of algebraic equations which can be solved easily. Illustrative examples and comparisons with Haar wavelets and Legendre wavelets are included to reveal the effectiveness of the method and the accuracy of the convergence analysis.

1. Introduction

Fractional calculus is generalized from integral calculus, it has been paid considerable attention since it plays a vital role in different scientific disciplines. Many natural phenomena can be modeled by fractional calculus, such as rheological behavior of soft magnetic elastomers, price volatility in finance, fast spreading of pollutants in hydrology, particle motions in heterogeneous environment and long particle jumps of the anomalous diffusion in physics, etc. [1–8]. As well as Fractional calculus, Fractional integro-differential equations (FIDEs) is generalized from integral integro-differential equations, and has superiority in easily modelling some natural physics processes and dynamic system processes. Due to the fractional order exponents in differential operators, analytical solutions of FIDEs are usually difficult to obtain. Consequently, many numerical methods have been developed to provide numerical solutions for FIDEs, such as fractional differential transform method [9,10], Adomian decomposition method [11], Taylor expansion approach [12], collocation method [13–15], Legendre Tau method [16], iteration method [17,18], Homotopy analysis method [19,20], wavelets method [21] and radial basis functions method [22]. However, the study on solving numerical solutions of systems of FIDEs has been paid less attention, there are only few papers like [23] considered numerical solutions for systems of FIDEs. In this paper, we adopt Bernoulli wavelets [24] to solve coupled systems of nonlinear fractional integro-differential equations.

Wavelets mean a family of piecewise functions, it can effectively approximate unknown function. The most used wavelets are Legendre wavelets [21], Chebyshev wavelets [25], and Haar wavelets [26]. Bernoulli wavelets as new constructed wavelets are first used to solve coupled systems of FIDEs. By combing Bernoulli wavelets function approximation with the fractional

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integral operational matrix which is different from that ones in [24], the studied systems of FIDEs can be transformed into easily solved systems of algebraic equations.

The main purpose of this paper is to use Bernoulli wavelets to solve coupled systems of FIDEs as

$$\begin{cases} D^{\alpha}u(t) = f_1(t, u(t), v(t)) + \int_0^t f_2(\tau, u(\tau), v(\tau))d\tau, \\ D^{\beta}v(t) = g_1(t, u(t), v(t)) + \int_0^t g_2(\tau, u(\tau), v(\tau))d\tau, \end{cases} \tag{1.1}$$

where $t, \tau \in [0, 1]$, $0 < \alpha, \beta \le 1$, D^{α} and D^{β} denote the Caputo derivative operator.

2. Preliminaries

Definition 2.1 ([21]). The Caputo derivative operator of order α is defined as

$$D^{\alpha}u(t) = \begin{cases} \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} u^{(n)}(\tau)d\tau, & \alpha > 0,\ n-1 < \alpha < n, \\ \frac{d^{(n)}u(t)}{dt^n}, & \alpha = n, \end{cases} \tag{2.1}$$

where $t \ge 0$, and $n \in \mathbb{N}$ ($\mathbb{N}$ denote positive integers). In this paper, only the case $0 < \alpha \le 1$ is considered.

Definition 2.2 ([27]). The Riemann–Liouville integral operator of order α is defined as

$$I^{\alpha}u(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} u(s)ds. \tag{2.2}$$

The Caputo derivative operator and Riemann–Liouville integral operator has the following properties

$$D^{\alpha}I^{\alpha}u(t) = u(t), \tag{2.3}$$

$$I^{\alpha}D^{\alpha}u(t) = u(t) - \sum_{k=0}^{n-1} \frac{u^{(k)}(0^+)}{k!} t^k, \qquad t \ge 0,\ n-1 < \alpha < n. \tag{2.4}$$

3. Function approximation using Bernoulli wavelets and Convergence analysis

3.1. Bernoulli wavelets

Bernoulli wavelets are defined on [0, 1] as follows [24]:

$$\psi_{nm}(t) = \begin{cases} 2^{\frac{k-1}{2}} \tilde{\beta}_m(2^{k-1}t - n), & \frac{n}{2^{k-1}} \le t \le \frac{n+1}{2^{k-1}}, \\ 0, & otherwise, \end{cases} \tag{3.1}$$

with

$$\tilde{\beta}_m(t) = \begin{cases} 1, & m = 0, \\ \frac{1}{\sqrt{\frac{(-1)^{m-1}(m!)^2}{(2m)!}\alpha_{2m}}} \beta_m(t), & m > 0, \end{cases} \tag{3.2}$$

for $n = 0, 1, \ldots, 2^{k-1} - 1$, $m = 0, 1, 2, \ldots, M-1$, $(k, M \in \mathbb{N})$, where $\beta_m(t)$ denote the Bernoulli polynomials with order m and can be defined on [0, 1] as

$$\beta_m(t) = \sum_{i=0}^{m} \binom{m}{i} \alpha_{m-i} t^i, \tag{3.3}$$

here $\alpha_i (i = 0, 1, 2, \ldots)$ are Bernoulli numbers, they come from the following formula

$$\frac{t}{e^t - 1} = \sum_{i=0}^{\infty} \alpha_i \frac{t^i}{i!}. \tag{3.4}$$