Applied Mathematics and Computation 323 (2018) 182–192

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Cascading crashes induced by the individual heterogeneity in complex networks

Jie Li[a,b], Juan Wang[c], Shiwen Sun[a,b], Chengyi Xia[a,b,*]

[a] *Tianjin Key Laboratory of Intelligence Computing and Novel Software Technology, Tianjin University of Technology, Tianjin 300384, PR China*
[b] *Key Laboratory of Computer Vision and System (Ministry of Education), Tianjin University of Technology, Tianjin 300384, PR China*
[c] *School of Electrical and Electronic Engineering, Tianjin University of Technology, Tianjin 300384, PR China*

A R T I C L E I N F O

Keywords:
System crash
Failure probability setup
Cascade size
Complex networks
Individual heterogeneity

A B S T R A C T

Deep understanding of the birth, growth and evolution of the real-life systems has been widely investigated, but the dynamics of system crashes are far beyond our knowledge. To this end, we propose a dynamical model to illustrate the collapsing behavior of complex networks, in which each node may leave the current networks since it has too few neighbors or has lost more than a specific proportion of its neighboring links. Different from previous works, the probability of being removed from the network for each node will be correlated with its original degree once the leaving conditions are satisfied, which includes the positive or negative correlation with the original degree, and totally independent probability deployment, and the individual heterogeneity has been integrated into these three probability setup schemes. Plenty of numerical simulations have indicated that the leaving probability setup scheme will greatly impact the system crashing behaviors under three different topologies including random, exponential and scale-free networks. In particular, the positively correlated scheme will substantially improve the survival of systems and further enhance the resilience of scale-free networks. To a great degree, the current results can help us to be further acquainted with the crashing dynamics and evolutionary properties of complex systems.

1. Introduction

Within the nature and human societies, almost all systems may undergo an evolutionary process, which includes the birth, growth and extinction, during their life cycle [1,2]. At present, many studies often focus on the emergence and evolution of complex systems, especially utilizing the approach from network science [3,4]. As an example, Watts and Strogatz developed a network model by rewiring some links in a ring lattice, which creates the famous small world effect simultaneously holding the shorter average distance and the larger clustering coefficient [5], theoretically illustrating the six degrees of separation in sociology; Barabási and Albert proposed the classical network generation model to elaborate the growth and evolution of many real-life systems and characterize the phenomenon of 'the rich becomes richer' by introducing the preference attachment [6]. These two seminal works triggered the extensive related researches regarding the complex

* Corresponding author at: Tianjin Key Laboratory of Intelligence Computing and Novel Software Technology, Tianjin University of Technology, Tianjin 300384, PR China.
E-mail addresses: juanwang75@163.com (J. Wang), xialooking@163.com (C. Xia).

https://doi.org/10.1016/j.amc.2017.11.059

networks [7–11] over the past two decades, and most works try to illustrate the structural evolution of many natural, social or artificial systems, and corresponding dynamics behavior taking place on top of complex networks. As an example, exploring the co-evolutionary cooperation behavior within the networked population has become an interdisciplinary topic [12–15]. Meanwhile, the epidemic spreading dynamics also have attracted a great deal of attention in the field of public hygienics, Internet security, online social networking analysis, opinion monitoring or controlling and so on [16,17]. Furthermore, mining the community properties hidden within many networked systems also helps to develop some valuable clues for the knowledge discovery and pattern recognition [18–20].

However, few studies dealt with the system crash and even disappearance process. Several typical works [21–23] involved the cascading process of complex networks and found that the scale-free (SF) networks exhibit the robust-yet-fragile properties, that is, SF networks are very robust against the random failures but become extremely fragile when they are subjected to the deliberate attacks. As a further step, interdependency among multiple networked systems (i.e., interdependent or interconnected networks) may surprisingly reduce the robustness against the random attacks since the failure of a node within one system may disable the function of its nearest neighboring nodes and corresponding ones within another coupling systems [24,25]. But these works often paid close attention to the steady state of attacked systems, and the crash dynamics during the attacking process is hardly concerned. In the meantime, there still exist some phenomena which are puzzling and need to be further understood. On the one hand, some outmoded systems may survive for long periods of time although they seem to be out of fashion and destined to be obsolescent. For instance, nearly each of Chinese feudal dynasties ranging from Qin to Qing, often lasts for over hundreds of years and they can still survive for tens of years even though they suffer from various rebellions and heavy blows before their crashes, but may suddenly collapse within one moment at a tipping point [26]; meanwhile, this state is often termed as the pseudo-steady one within the academia. On the other hand, some systems are thought to be seemingly large or strong enough, but may crash rapidly. Examples include the sudden collapse of some biological [27] and ecological [28] systems, which is sometimes faster than expected or even without any early symptom [29,30]. In the human society, the abrupt crashes of previous Soviet Union and some Arabian countries including Egypt and Libya are also beyond our imagination [31]. Moreover, the Friendster online social network [32], which was once the biggest one in the world, quickly lost its most of users within several months as a result of a minor and unfriendly interface revision. Thus, in order to deeply understand the collapsing mechanism of many natural, biological, ecological, social, physical or cyber-physical systems and even human society, it is necessary for us to develop the new framework and approach used for modeling the evolutionary and crashing dynamics of complex systems.

So far, several recent works have begun to explore the related problems to help accounting for some respects of system crashes or the crashing dynamics under some extreme cases [33,34]. Among them, the studies on the herd behavior, which concern the individual decision character within a group without any centralized control protocol, provide some useful hints to explain how the individuals collectively act without any global information. Taking an example, the herd-effect theory, which discusses the potential mechanism to create the collective and consistent operations [35], may be beneficial to expound the sharp change when most individuals lie over the borderline between multiple possible alternatives. In addition, information cascade theories on top of social or online social networks [36] have found that a minor shock may lead to the major shift provided that the individuals make the choice just based on the strategies from their local and neighboring agents, that is, the global cascade may suddenly emerge and phase transitions occur when the specified conditions are satisfied. The third aspect related with the system crash is about the studies of lock-in effects and switching cost, which estimates the impact of cost on the cascade behaviors when an individual decides to change from one choice or strategy to the other one.

All these researches, however, cannot comprehensively evaluate the system crash dynamics within complex networking population, and exploring the structural evolution of complex networks and dynamical properties on top of them has become a very important topic in the field of complex systems in the recent years. Two notable and relevant works that simultaneously consider the system crash dynamics and networking topology are both based on the k-core cascade theory, in which the k-core of a network denotes its maximal subset comprising all nodes with degree equal to or greater than k [37]. Meanwhile, it is often assumed that a network node with fewer than k links will be removed from the current network with a specific probability, and hence measuring the k-core of a network can estimate the size of residual giant component when the value of k is increased due to various reasons. Nevertheless, k-core theory can reproduce the sudden crashing process of systems, in particular when there exists a big jump for the value of k, but the pseudo-steady state for the real-world systems cannot come forth. To this end, Yu et al. [38] propose a complex network-based model to characterize the process and dynamics of a system crash and also the pseudo-steady state under some cases. In this model, each node may leave the current network with a specific probability on account of two possible reasons: one is that the focal node degree (k) is less than the specified core number(k_s), which means that this node cannot gain the sufficient support or benefit to stay on; the other one is that the node has lost more than a specific proportion (denoted by q) of its original neighbors, which implies that the focal node may leave the current network in order to reduce the risk of becoming a victim or join another system for a more promising future. For the brevity, this model can be termed as KQ-cascade model in what follows, which is different from the k-core based model presented in Ref. [32,37], and also substantially differs from the traditional threshold model [36,39], which often assumes that nodes leave only because they lose q fraction of nearest neighbors and especially concentrates on analyzing the role of threshold value q in the process initiating the global cascade. Large quantities of simulations have been conducted to demonstrate that the theoretical analyses developed for KQ-cascade model can well predict the system crashing process and even reproduce the pseudo-steady state in some real-life online social networks under the