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Collocation method based on rational Legendre functions for solving the magneto-hydrodynamic flow over a nonlinear stretching sheet

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ABSTRACT

In this paper, a direct collocation method based on rational Legendre functions is proposed for solving the magneto-hydrodynamic (MHD) boundary layer flow over a nonlinear stretching sheet. Here, we use rational Legendre–Gauss–Radau nodes and transformed Hermite–Gauss nodes as interpolation points. We present the comparison of this work with some other numerical results. Moreover, residual norm shows that the present solutions are accurate and applicable.

1. Introduction

In the last two decades, different spectral methods have been proposed for solving problems on unbounded domains [1,2]. A brief review on some of the recent advances in the spectral methods for unbounded domains is presented in [3]. An effective method that is used to solve problems in semi-infinite domains is based on rational orthogonal functions, such as the rational Legendre functions and rational Chebyshev functions [3–6]. There are several reasons to approximate by rational orthogonal functions. Firstly, they are easily implemented and give good accuracy [5]. Secondly, as mentioned in [7], their weights are much weaker than the Hermite and Laguerre functions and so it is not usually required to reform the original problems. The rational orthogonal functions are widely used for solving a wide range of problems in semi-infinite domains including, Lane–Emden equation [8], laminar boundary layer flow [9], Volterra's population model [10], Thomas–Fermi problem [11] and Burgers equation [7]. The interested reader can see [12–20] for more research works in the solution of problems in semi-infinite domains.

In this paper, we investigate the problem of the boundary layer flow of an incompressible viscous fluid over a non-linear stretching sheet. This type of flow, has been studied by several researchers. Hayat et al. [21] used the modified Adomian decomposition method and Rashidi [22] employed the modified differential transform method. Also, homotopy perturbation method [23–25], homotopy analysis method [26], spectral homotopy analysis method [27], homotopy perturbation transform method [28], rational Chebyshev collocation method [29] and optimal homotopy asymptotic method [30] are used to solve this problem.

The purpose of this paper is to solve this problem using the rational Legendre collocation method. Here, we use rational Legendre–Gauss–Radau nodes and transformed Hermite–Gauss nodes as interpolation points. Collocation methods are a

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nice and powerful approach for numerical solution of nonlinear physical problems, chiefly because they offer the simplest treatment of nonlinear terms [31–36].

The organization of the rest of this article is as follows. In the next section, we give the problem formulation and in Section 3, we explain some properties of rational Legendre functions and Hermite polynomials required for our subsequent development. In Section 4, we apply the rational Legendre collocation method using rational Legendre–Gauss–Radau points and transformed Hermite–Gauss points on the problem. Section 5 shows the approximate solutions and compares them with other findings. Section 6 is devoted to conclusions.

2. Problem formulation

Consider the MHD flow of an incompressible viscous fluid over a stretching sheet at $y=0$. The fluid is electrically conducting by the influence of an applied magnetic field $B(x)$. We assume that $B(x)$ is applied normal to the direction of the flow and the induced magnetic field is neglected. The governing continuity and momentum equations (see [21,23,26,27]) are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2(x)}{\rho}u. \tag{2}$$

Here u and v, respectively, represent the velocity components in the x and y directions, with the x-axis chosen along the stretching sheet, ρ is the fluid density, ν is the kinematic viscosity and σ is the electrical conductivity parameter of the fluid. In Eq. (2) the external electric field and the polarization outcomes are negligible and [21,37]

$$B(x)=B_0x^{\frac{n-1}{2}},$$

where B_0 is the strength of the magnetic field. The boundary conditions corresponding to the non-linear stretching of a sheet are

$$u(x,0)=cx^n,\qquad v(x,0)=0,$$

$$u(x,y)\to 0,\quad \text{as}\quad y\to\infty, \tag{3}$$

where c and n are constants. By applying the transformations

$$u=cx^n f'(z),\quad \text{where}\quad z=\sqrt{\frac{c(n+1)}{2\nu}}x^{\frac{n-1}{2}}y,$$

$$v=-\sqrt{\frac{c\nu(n+1)}{2}}\left[f(z)+\frac{n-1}{n+1}z f'(z)\right]x^{\frac{n-1}{2}},$$

Eqs. (1) and (2) are transformed to the nonlinear differential equation

$$f'''(z)+f(z)f''(z)-\beta f'^2(z)-M f'(z)=0, \tag{4}$$

with boundary conditions

$$f(0)=0,\qquad f'(0)=1,\qquad f'(\infty)=0, \tag{5}$$

where $\beta=\dfrac{2n}{1+n}$ and $M=\dfrac{2\sigma B_0^2}{\rho c(1+n)}$ are stretching and magnetic parameters respectively. It is worth indicating that, for a linear stretching sheet (that is $n=\beta=1$), the exact analytical solution of (4) and (5) is (see [27] and references therein)

$$f(z)=\frac{1-\exp(-\sqrt{1+M}z)}{\sqrt{1+M}}. \tag{6}$$

In this case $f''(0)=-\sqrt{1+M}$ and this solution is used to validate the numerical method.

3. Properties of rational Legendre functions and Hermite polynomials

In this section, we briefly present some basic features of rational Legendre functions induced by the Legendre polynomials and later we present some properties of Hermite polynomials.