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Robust disturbance rejection for uncertain fractional-order systems

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ABSTRACT

This paper describes a disturbance rejection scheme that adopts equivalent-input-disturbance (EID) approach for uncertain fractional-order (FO) systems. An EID estimator that contains an FO observer is designed to actively compensate for the disturbances and process modeling uncertainties without requiring their prior knowledge. Under the construction of the FO control system, a robust stability condition and the parameters of the controller are derived using a linear matrix inequality based method. Finally, numerical and practical examples are illustrated to demonstrate the validity and superiority of the method.

1. Introduction

Fractional-order (FO) calculus, as a pure mathematical theory, has been in use for approximately three centuries. However, it was not applied to control systems until FO derivatives and integrals provided a powerful instrument for describing the hereditary properties of different systems. The control systems in the physical world are FO in nature. In the past decades, FO control technique has been applied to solve several practical problems, such as motor control, signal processing, and control of autonomous vehicles [1–5]. These FO models more accurately demonstrate the dynamic processes and properties of actual systems than integer-order models do.

FO controllers always exhibit better control performance for fractional-order system (FOS) control than integer-order ones [6]. Four main kinds of FO controllers exist, namely, CRONE, TID, and FO PID controllers and FO lead-lag compensators [6–8]. On the basis of stability theories of the FOS (e.g., [9–13]), researchers have recently developed several advanced techniques for FOS control system design. By introducing an appropriate switching surface, the well-known sliding mode control has been applied to uncertain and nonlinear FOSs [14,15]. Lan et al. presented a robust observer-based control method using indirect Lyapunov approach [16,17]. Adaptive control has been successfully used for FOSs with non-commensurate orders by designing an adaptive backstepping controller [18]. For model independence, easy parameter tuning, and strong robustness, active disturbance rejection control (ADRC) provides an alternative for FOS control, such as those of nonlinear FOSs and FO chaotic systems [19,20].

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The disturbance rejection problem needs to be addressed in the design of control systems. The disturbances in the FOS are difficult to reject due to the complexity of FO differential equations [21,22]. The disturbance rejection problem for FOSs has not been considered in most of the existing studies. In [23], ADRC was first used for the FOS, where the closed-loop system is treated as a second-order system, FO is regarded as a part of the total disturbance, and an external state observer is used to compensate for the disturbances. Given that the design does not focus on the FO itself, it would cause a major model error and require a high observer bandwidth for accurate state estimation. Li et al. developed a fractional ADRC method using the fractional external state observer, which estimates the total disturbance and the FO dynamic states [24]. However, the uncertainties have not been mentioned. Periodic disturbance is considered on the basis of an adaptive orthogonal signal generator, which permits the reconstruction of an unknown disturbance and the cancellation of its effect on the system output [25].

This paper focuses on the disturbance rejection of an FOS with uncertainties. The equivalent-input-disturbance (EID) approach is effective for the disturbance rejection of integer-order uncertain systems [26,27]. Motivated by it, the uncertain FO control system is designed applying the EID method. The configuration of the control system involves an FO state observer and a low-pass filter. This method compensates for external disturbances and uncertainties of the FOS without the need for any information about the disturbances. A robust stability condition of the control system and the design of the parameters are derived using linear matrix inequality (LMI) technique. This stability condition is available for order $0 < \alpha < 2$ and is less conservative for $1 \le \alpha < 2$. Simulations demonstrate that the presented method effectively rejects disturbances and handles uncertainties for an FOS.

2. Preliminaries

Throughout this paper, we denote by $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ an n-dimensional Euclidean space and the set of all $m \times n$ real matrices, respectively. The notation $A > 0$ ($A < 0$) with A being a symmetric matrix means that the matrix A is positive (negative) definite. I and 0 mean the identity matrix of appropriate dimension and the null matrix of appropriate dimension. X^T stands for the transpose of X. We denote by sym(X) the expression $X^T + X$. The notation $\begin{bmatrix} X & Y \\ * & Z \end{bmatrix}$ stands for $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix}$.

In this paper, the Caputo fractional derivative definition is used. This is because the initial conditions for fractional differential equations with Caputo derivative take on the same form as those for integer-order ones. Referring to [7], the Caputo fractional derivative of the function $f(t)$ with starting point $t_0 = 0$ is defined by

$$ {}_0D_t^{\alpha} f(t) = \frac{1}{\Gamma(a-m)} \int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha+1-m}} d\tau, $$

where $\Gamma(\cdot)$ is the well-known Gamma function, $\Gamma(z) = \int_0^\infty e^{-z} t^{z-1} dt$, and m is an integer satisfying $m - 1 < \alpha \le m$.

In this section, the following lemmas are introduced, which will be used in the control design of the FOS in the next section.

Lemma 1. *[28] Let $A \in \mathbb{R}^{n\times n}$ be a real matrix. Then the FOS ${}_0D_t^{\alpha} x(t) = Ax(t)$, where $1 \le \alpha < 2$, is asymptotically stable, that is $|\arg(specA)| > \alpha\pi/2$, if and only if there exists a symmetric matrix $P > 0$ such that*

$$ \begin{bmatrix} (AP + PA^T)\sin\theta & (AP - PA^T)\cos\theta \\ * & (AP + PA^T)\sin\theta \end{bmatrix} < 0, $$

where $\theta = \pi - \alpha\pi/2$.

Lemma 2. *[29] For a given matrix $\Pi \in \mathbb{R}^{p\times n}$ with $\mathrm{rank}(\Pi) = p$, there exists a matrix $\bar{X} \in \mathbb{R}^{p\times p}$ such that $\Pi X = \bar{X}\Pi$ holds for any $X \in \mathbb{R}^{n\times n}$, if and only if X can be decomposed as*

$$ X = W \begin{bmatrix} \bar{X}_{11} & 0 \\ 0 & \bar{X}_{22} \end{bmatrix} W^T, $$

where $W \in \mathbb{R}^{n\times n}$ is an orthogonal matrix, $\bar{X}_{11} \in \mathbb{R}^{p\times p}$, and $\bar{X}_{22} \in \mathbb{R}^{(n-p)\times(n-p)}$.

Lemma 3. *[30] Given matrices Y, D and E of appropriate dimensions. $Y + DFE + E^T F^T D^T < 0$ holds for all F satisfying $FF^T < I$, if and only if there exists an $\varepsilon > 0$ such that $Y + \varepsilon DD^T + \varepsilon^{-1} E^T E < 0$.*

Lemma 4 (Schur complement)**.** *[31] For a given symmetric matrix*

$$ \Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12}^T & \Sigma_{22} \end{bmatrix}, $$

the following statements are equivalent:

(1) $\Sigma < 0$;
(2) $\Sigma_{11} < 0$ *and* $\Sigma_{22} - \Sigma_{12}^T \Sigma_{11}^{-1} \Sigma_{12} < 0$;
(3) $\Sigma_{22} < 0$ *and* $\Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{12}^T < 0$.