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On solving systems of multi-pantograph equations via spectral tau method

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ABSTRACT

The current manuscript focuses on solving systems of multi-pantograph equations. The spectral tau method is applied for solving systems of multi-pantograph equations with shifted Jacobi polynomials as basis functions. The convergence analysis of the proposed technique is also investigated. We introduced the numerical solutions of some test problems and compared the obtained numerical solutions of such problems with those given using different numerical methods.

1. Introduction

As an important kind of delay differential equations, the pantograph equation has been used in modeling many real-life phenomena, such as economy, biology, astrophysics, control and electrodynamics, see [1–3]. Various numerical approaches were utilized for pantograph equations; Bota and Căruntu [4] applied the ϵ-Approximate polynomial method for solving multi-pantograph equations with variable coefficients, Akkaya et al. [5] applied a numerical approach based on the First Boubaker polynomials for solving pantograph delay differential equations, Doha et al. [6] applied the Jacobi rational-Gauss collocation method for solving generalized pantograph equations, Ahmed and Mukhtar [7] applied a stochastic approach for solving multi-pantograph equations, Cheng et al. [8] applied the reproducing kernel method for neutral functional-differential equation, Reutskiy [9] used the spectral collocation method for approximating the solution of pantograph functional differential equations, Akyuz-Dascioglu and Sezer [10] applied the Bernoulli collocation method for solving high-order generalized pantograph equations and Javadi et al. [11] introduced a numerical method based on Bernstein polynomials to solve generalized pantograph equations.

As a spectral approach for solving numerically some kinds of differential equations, the tau method is classified as one of the most important used methods due to its high convergence and accuracy. Saadatmandi and Dehghan [12] used the tau method for the one-dimensional parabolic inverse problem, while the authors in [13,14] applied the tau approximation for solving the weakly singular Volterra integral equation. Soltanalizadeh et al. [15] introduced a new method based on the tau approximation for the Sobolev-type differential equation with nonlocal boundary conditions. In [16], the authors used the Legendre wavelets as basis of the spectral tau approximation to solve integral equations, while Pishbin [17] applied an operational tau technique for solving systems of integro-differential equations. Recently, Shao et al. [18] used the Chebyshev polynomials as basis of the tau approach with the domain decomposition method for solving singular perturbed problems. The spectral tau approach is applied based on shifted Jacobi polynomials for solving various kinds of fractional differential equations, see [19–22].

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In Koroma et al. [23], the decomposition approach is used with the Laplace transform method to solve the system of multi-pantograph equations, while Sedaghat et al. [24] introduced a different numerical technique based on Chebyshev polynomials with operational matrix method for getting its numerical solution. Also, Yüzbasi [25] used the first kind of Bessel functions, the spectral collocation technique and the operational matrix method for the system of multi-pantograph equations.

Here, we intend to develop a numerical technique for solving the system of multi-pantograph equations:

$$\sum_{s=1}^{l} a_{p,s}u_s^{(1)}(t) = \sum_{s=1}^{l} b_{p,s}u_s(t) + \sum_{r=1}^{m}\sum_{s=1}^{l} c_{p,s}^{r}u_s(q_r t) + g_p(t), \qquad 1 \le p \le l, \quad 0 \le t \le T, \tag{1.1}$$

subject to

$$u_p(0) = d_p, \qquad p = 1, 2, \dots, l, \tag{1.2}$$

where $a_{p,s},\ b_{p,s},\ c_{p,s}^{r},\ q_r,\ d_p\ (1 \le p, s \le l),\ 0 < q_r < T\ (r = 1, 2, \dots, m)$ are real numbers with $a_{p,\,p} \neq 0$, $u_s(t) (1 \le s \le l)$ are the unknown functions we have to find, and $g_p(t) (1 \le p \le l)$ are known functions defined in [0, T]. For achieving our main goal, we utilize the shifted Jacobi polynomials as basis of the spectral tau approach to convert the system of multi-pantograph equations (1.1) and (1.2) into a system of algebraic equations.

Our manuscript is outlined as follows: In the following section, we present some relevant properties of shifted Jacobi polynomials, while in Section 3, we develop a numerical method for solving the system of multi-pantograph equations (1.1) and (1.2) based on the spectral tau approach. The convergence analysis of the proposed numerical approach is presented in Section 4. In Section 5, we test the proposed approach on two numerical examples. Conclusions are given in Section 6.

2. Shifted Jacobi polynomials

Assuming that the Jacobi polynomial of degree i is denoted by $P_i^{(\gamma,\delta)}(x);\ \gamma \ge -1,\ \delta \ge -1$ (defined on the interval $(-1, 1)$). Then $P_i^{(\gamma,\delta)}(x)$ may be generated by the recurrence formula:

$$P_{i+1}^{(\gamma,\delta)}(x) = (\phi_i x - \varphi_i)P_i^{(\gamma,\delta)}(x) - \psi_i P_{i-1}^{(\gamma,\delta)}(x), \qquad i \ge 1,$$

with

$$P_0^{(\gamma,\delta)}(x) = 1, \quad P_1^{(\gamma,\delta)}(x) = \frac{1}{2}(\gamma+\delta+2)x + \frac{1}{2}(\gamma-\delta),$$

where

$$\begin{aligned}
\phi_i &= \frac{(2i+\gamma+\delta+1)(2i+\gamma+\delta+2)}{2(i+1)(i+\gamma+\delta+1)},\\
\varphi_i &= \frac{(2i+\gamma+\delta+1)(\delta^2-\gamma^2)}{2(i+1)(i+\gamma+\delta+1)(2i+\gamma+\delta)},\\
\psi_i &= \frac{(2i+\gamma+\delta+2)(i+\gamma)(i+\delta)}{(i+1)(i+\gamma+\delta+1)(2i+\gamma+\delta)}.
\end{aligned}$$

Introducing the change of variable $x = \frac{2t}{T} - 1$, then we get the shifted Jacobi polynomials $(P_{T,i}^{(\gamma,\delta)}(t))$ defined on the interval (0, T) that may be generated by:

$$P_{T,i+1}^{(\gamma,\delta)}(t) = (\theta_i t - \vartheta_i)P_{T,i}^{(\gamma,\delta)}(t) - \psi_i P_{T,i-1}^{(\gamma,\delta)}(t), \quad i \ge 1,$$

with

$$P_{T,0}^{(\gamma,\delta)}(t) = 1, \quad P_{T,1}^{(\gamma,\delta)}(t) = \frac{1}{T}(\gamma+\delta+2)t - (\delta+1),$$

where

$$\begin{aligned}
\theta_i &= \frac{(2i+\gamma+\delta+1)(2i+\gamma+\delta+2)}{T(i+1)(i+\gamma+\delta+1)},\\
\vartheta_i &= \frac{(2i+\gamma+\delta+1)(2i^2+(1+\delta)(\gamma+\delta)+2i(\gamma+\delta+1))}{(i+1)(i+\gamma+\delta+1)(2i+\gamma+\delta)}.
\end{aligned}$$

These polynomials are satisfying the orthogonality relation [26]

$$\int_0^T P_{T,i}^{(\gamma,\delta)}(t)P_{T,j}^{(\gamma,\delta)}(t)w_T^{(\gamma,\delta)}(t)dt = h_{T,j}^{(\gamma,\delta)}, \tag{2.1}$$