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Decay-rate-dependent conditions for exponential stability of stochastic neutral systems with Markovian jumping parameters

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ABSTRACT

This note studies the problem of decay-rate-dependent exponential stability for neutral stochastic delay systems with Markovian jumping parameters. First, by introducing an operator $\mathcal{D}(x_t, i)$ as well as a novel Lyapunov–Krasovskii functional, sufficient conditions for exponential stability of system with a decay rate are obtained. Second, the results are extended to the robust exponential estimates for uncertain neutral stochastic delay systems with Markovian jumping parameters. Finally, numerical examples are provided to show the effectiveness of the proposed results.

1. Introduction

Neutral stochastic differential delay equations (NSDDEs) are often used to describe some practical dynamical systems, such as lossless transmission systems, chemical systems and neural networks, see [1,2]. The literature of studying the existence, uniqueness and continuous dependence of the solution for NSDDEs can be found in [1]. The problems of stability analysis, stabilization and control for neutral stochastic delay systems (NSDSs) have attracted considerable attention in the past few decades, see, for example, [3–13], and references therein.

On the other hand, as a result of component failures or repairs, changing subsystem interconnections, and abrupt environmental disturbances, many practical systems may experience abrupt variations in their structure. To model such systems, neutral stochastic differential delay equations with Markovian switching parameters have been developed, see [14–17], It should be stressed that the fundamental theory of the existence and uniqueness of the solution to NSDDEs with Markovian switching was established in [14], where the asymptotic boundedness as well as the exponential stability in pth moment are also discussed. Shortly afterwards, almost surely asymptotic stability of NSDDEs with Markovian switching was investigated in [15,17]. Stability in distribution of NSDDEs with Markovian switching was considered in [16]. The stabilization problem of neutral stochastic delay Markovian jump systems by state feedback controllers was addressed in [18]. Furthermore, further stability analysis and application of NSDDEs with Markovian switching have been given in [19–27].

An interesting issue in the application of systems control is the exponential estimate. The objective of the exponential estimate problem for delay systems is to derive exponential stability conditions depending on the decay rate, which enables the designers to check whether the system is exponentially stable with a prescribed decay rate. Recently, a great number

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of significant results on the exponential estimate of delay systems have been reported in the literature. For example, the exponential estimate of neutral time delay systems was investigated in [28]. By introducing slack variables, new exponential estimate for time delay systems was addressed in [29]. The exponential estimate of uncertain linear Markovian jump systems was proposed in [30]. Using the method of delay partitioning, the exponential estimate problem of Markovian jump systems with mode-dependent time varying delays was considered in [31].

Motivated by the previous results related to time delay systems, in this paper we are concerned with the exponential estimate of neutral stochastic delay systems (NSDSs) with Markovian jumping parameters. For time delay systems, slack variables are usually introduced in stability results to reduce conservatism, see [5,7,32–38]. More narrowly, slack variables, the Jensen's inequality and matrix-refined-function in these literatures have been used to handle the derivative of the double integral term $\int_{-\bar{\tau}}^{0}\int_{t+\theta}^{t}\dot{x}(s)^T Z\dot{x}(s)\mathrm{d}s\mathrm{d}\theta (Z>0)$ in Lyapunov–Krasovskii(L–K) functional. However, this term does not appear in L–K functional when we study the stochastic system. In addition, the neutral delay and Markovian jumping parameters also influence stability and performance index of the system. Thus, these techniques can not be applied in stability analysis and control design of the system model, when the Markovian jumping parameters, stochastic disturbances and neutral delay appear simultaneously. Consequently, we need to put forward methods to overcome these difficulties. The key contribution of this work is to derive some sufficient conditions of exponential estimates of NSDSs with Markovian jumping parameters by introducing the neutral operator $\mathcal{D}(x_t, i)$ as well as an augmented Lyapunov–Krasovskii functional.

Notation: The notation $\mathbb{R}^n$ denotes the n-dimensional Euclidean space with the Euclidean norm $|\cdot|$. For a real matrix M, M^T represents the transpose of M, and $\|M\|$ denotes the spectral norm of matrix M. I denotes an identity matrix with an appropriate dimension. Assume that X and Y are real symmetric matrices, the notation $X>Y$ means that the matrix $X-Y$ is positive definite. In a symmetric matrix, the symmetric terms are denoted by $*$. $a\vee b$ denotes the maximum of a and b, $a\wedge b$ denotes the minimum of a and b. Assume that Ω is a sample space, $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ denotes a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$, where $\mathcal{F}$ is the σ-algebra of subset of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. $\mathbb{E}(\cdot)$ is the expectation operator with respect to the probability measure $\mathcal{P}$. Let $\tau>0$, the family of all continuous $\mathbb{R}^n$-valued function on $[-\tau, 0]$ is denoted by $C([-\tau, 0];\ \mathbb{R}^n)$. $C^b_{\mathcal{F}_0}([-\tau, 0];\ \mathbb{R}^n)$ denotes the family of all $\mathcal{F}_0$-measurable bounded $C([-\tau, 0];\ \mathbb{R}^n)$-valued random variables $\xi = \{\xi(\theta) : -\tau \leq \theta \leq 0\}$.

2. Problem formulation

Consider the following neutral stochastic delay system with Markovian jumping parameters:

$$\mathrm{d}[x(t) - D(r_t)x(t-\tau_1)] = [A(r_t)x(t) + A_1(r_t)x(t-\tau_2)]\mathrm{d}t + [C(r_t)x(t) + C_1(r_t)x(t-\tau_2)]\mathrm{d}B(t), \tag{1}$$

$$x(t) = \varphi(t), \quad \forall t \in [-\bar{\tau},\ 0], \quad \bar{\tau} = \tau_1 \vee \tau_2, \tag{2}$$

where $x(t)\in\mathbb{R}^n$ is the system state, τ_1 and τ_2 denote the neutral delay and discrete delay, $D(r_t)$, $A(r_t), A_1(r_t)$, $C(r_t), C_1(r_t)$ are system matrices with the Markov jump process $\{r_t, t\geq 0\}$. Here, $\{r_t, t\geq 0\}$ is a continuous time Markov process with right continuous trajectories taking values in $\mathcal{S} = \{1, 2, \ldots, N\}$. The transition probabilities is given by

$$\Pr\{r_{t+\Delta t} = j \mid r_t = i\} = \begin{cases} \gamma_{ij}\Delta t + o(\Delta t), & i\neq j \\ 1+\gamma_{ii}\Delta t + o(\Delta t), & i = j, \end{cases}$$

where $\Delta t>0$, and $\gamma_{ij}\geq 0$, for $i\neq j$, is the transition rate from mode i at time t to mode j at time $t+\Delta t$, $\gamma_{ii} = -\sum_{j=1, j\neq i}^{N}\gamma_{ij}$, $\lim_{\Delta t\to 0}(o(\Delta t)/\Delta t) = 0$; $B(t)$ is a scalar Brownian motion defined on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ satisfying $\mathbb{E}\{\mathrm{d}B(t)\} = 0$, $\mathbb{E}\{\mathrm{d}B^2(t)\} = \mathrm{d}t$, and we assume that $B(t)$ is independent from the Markov chain $\{r_t,\ t\geq 0\}$. To simplify the notation, for each $r_t = i$, $i\in\mathcal{S}$, $A(r_t)$, $A_1(r_t)$ will be denoted by A_i and A_{1i}. Denote $\kappa = \max_{i\in\mathcal{S}}\|D_i\|$, and we assume that $k\in(0,\ 1)$. The purpose of this paper is to obtain sufficient conditions of exponential estimates on systems (1) and (2). To this end, we need the following definition.

Definition 1. ([29]) Systems (1) and (2) is said to be exponentially stable in mean square with a decay rate λ if there exist scalars $\sigma>0$ and $\lambda>0$ such that

$$\mathbb{E}|x(t)|^2 \leq \sigma e^{-\lambda t}|\varphi|^2_{\bar{\tau}}, \tag{3}$$

where $|\varphi|_{\bar{\tau}} = \sup_{-\bar{\tau}\leq t\leq 0}|\varphi(t)|$.

3. Exponential stability

To begin with, an exponential estimate of systems (1) and (2) in the form (3) for $\tau_1\neq\tau_2$ will be considered in this section.

Theorem 3.1. *Choose three positive numbers $\lambda>0$, $\tau_1>0$ and $\tau_2>0 (\tau_1\neq\tau_2)$ such that*

$$\lambda < \frac{1}{\bar{\tau}}\log\frac{1}{\max_{i\in\mathcal{S}}\|D_i\|}, \quad \bar{\tau} = \tau_1\vee\tau_2. \tag{4}$$