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Comparison among unstructured TVD, ENO and UNO schemes in two- and three-dimensions

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ABSTRACT

This study focuses on unstructured TVD, ENO and UNO schemes applied to solve the Euler equations in two- and three-dimensions. They are implemented on a finite volume context and cell centered data base. The algorithms of Yee, Warming and Harten 1982; Harten; Yee and Kutler; Yee Warming and Harten 1985; Yee; Yee and Harten; Harten and Osher; Yang 1990, Hughson and Beran; Yang 1991; and Yang and Hsu are implemented to solve such system of equations in two- and three-dimensions. All schemes are flux difference splitting and good resolution is expected. This study deals with calorically perfect gas model and in so on the cold gas formulation has been employed. Two problems are studied, namely: the transonic convergent-divergent symmetrical nozzle, and the supersonic ramp. A spatially variable time step is implemented to accelerate the convergence process. The results highlights the excellent performance of the Yang 1990 TVD scheme, yielding an excellent pressure distribution at the two-dimensional nozzle wall, whereas the Harten and Osher scheme yields accurate values to the angle of the oblique shock wave and the best wall pressure distributions in the two-dimensional ramp problem. On the other hand, the excellent performance of the Harten scheme in the three-dimensional nozzle problem, yielding an excellent pressure distribution at the nozzle wall, and the Yee and Harten scheme yielding an accurate value to the angle of the oblique shock wave and the best wall pressure distribution in the three-dimensional ramp problem are of good quality.

1. Introduction

High resolution upwind schemes have been developed since 1959, aiming to improve the generated solution quality, yielding more accurate solutions and more robust codes. The high resolution upwind schemes can be of flux vector splitting type or flux difference splitting type. In the former case, more robust algorithms are yielded, while in the latter case, more accuracy is obtained. Several studies were performed involving high resolution TVD algorithms in the international literature, as for example:

The scheme of Yee et al. [1] is a flux difference splitting one, which utilizes the concept of TVD ("Total Variation Diminishing"). It utilizes the concept of a modified inviscid flux vector to calculate the numerical fluxes. It utilizes artificial

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compressibility terms to take into account compressibility effects. The second-order of accuracy is obtained with an appropriate definition of the modified numerical flux. The scheme satisfies a proper entropy inequality to ensure that the limit solution will have only physically relevant discontinuities. This scheme is second-order accurate in space and time. The time integration is accomplished by a Runge–Kutta procedure in this work.

The work of Harten [2] defined a class of new explicit second-order accurate finite difference schemes, for the computation of weak solutions of hyperbolic conservation laws. The highly non-linear schemes were obtained by applying a non-oscillatory first-order accurate scheme to an appropriately modified flux function. The so-derived second-order accurate schemes achieve high resolution while preserving the robustness of the original non-oscillatory first-order accurate scheme. These schemes are called TVD ("Total Variation Diminishing") ones and yield physically relevant solutions by the use of an entropy condition. Our implementation of the Harten's [2] scheme is accomplished on a finite volume context. This scheme is second-order accurate in space. The time integration is accomplished by a Runge–Kutta method of two stages.

The study of Yee and Kutler [3] emphasized that a one-parameter family of explicit and implicit second-order accurate, entropy satisfying, total variation diminishing (TVD) schemes had been developed by Harten [2]. These schemes had the property of not generating spurious oscillations for one-dimensional non-linear scalar hyperbolic conservation laws and constant coefficient hyperbolic systems. Application of these methods to one- and two-dimensional fluid flows containing shocks (in Cartesian coordinates) yielded highly accurate non-oscillatory numerical solutions. The authors extended these methods to the multidimensional Euler equations in a generalized coordinate systems. The scheme is second-order accurate in space and time. The time integration is accomplished by a Runge–Kutta method of two stages, second-order accurate.

A new implicit unconditionally stable high resolution TVD scheme to steady state calculations was presented by Yee et al. [4]. It comprises a member of a one-parameter family of explicit and implicit second-order accurate schemes developed by Harten [2] for the computation of weak solutions of one-dimensional hyperbolic conservation laws. The scheme was guaranteed not to generate spurious oscillations for a nonlinear scalar equation and a constant coefficient system. Numerical experiments have shown that the scheme not only had a fairly rapid convergence rate, but also generated a highly resolved approximation to the steady state solution. A detailed implementation of the implicit scheme for the one- and two-dimensional compressible inviscid equations of gas dynamics was presented. Some numerical experiments of one- and two-dimensional fluid flows containing shocks demonstrated the efficiency and accuracy of the new scheme.

The scheme of Yee [5] reformulates a one-parameter family of second-order explicit and implicit total variation diminishing (TVD) algorithms so that a simpler and wider group of limiters was included. The resulting scheme can be viewed as a symmetrical algorithm with a variety of numerical dissipation terms that were designed for weak solutions of hyperbolic problems. This was a generalization of the works of Roe and Davis to a wider class of symmetric schemes other than Lax-Wendroff. The main properties of this class of schemes were that they could be implicit, and, when steady-state calculations were sought, the numerical solution was independent of the time step. Numerical experiments with two-dimensional unsteady and steady-state airfoil calculations have shown that the proposed symmetric TVD schemes were quite robust and accurate. In the present study, only the results with the Minmod limiter, Eq. (37), are presented.

The work of Yee and Harten [6] considered that Harten's method of constructing high resolution TVD schemes involves starting with a first-order TVD scheme and applying it to a modified flux approach. The modified flux is chosen so that the scheme is second-order at regions of smoothness and first-order at points of extrema. This technique is sometimes referred to as the modified flux approach. The authors extended a TVD scheme (via the modified flux approach) to generalized coordinate systems and discussed the various solution strategies for the implicit TVD schemes for more efficient two-dimensional steady-state applications. The TVD scheme was initially an implicit TVD one developed to solve a two-dimensional gas-dynamic problem in Cartesian coordinate.

Considering the axisymmetric configurations, [7] Hughson and Beran presented an explicit, second-order accurate, total variation diminishing (TVD) scheme applied to the Euler equations in axisymmetric form to study hypersonic blunt-body flows. The modified flux function approach of Harten [2], with modification by Yee [8], for two-dimensional flows is extended to treat axisymmetric flows. The scheme was presented on a finite difference context, but in our implementation, the scheme is written on a finite volume context. Roe's averaging for a perfect gas was used to assess the eigenvalues and eigenvectors at cell interfaces, because it has the computational advantage of resolving stationary discontinuities. An entropy condition is implemented to assure relevant physical solutions. Linear and non-linear limiters (g's functions) assure second-order accuracy as well control the amount of numerical dissipation added to the flow equations.

Recently, a new class of uniformly high order accurate essentially nonoscillatory (ENO) schemes have been developed by [9–12]. They presented a hierarchy of uniformly high order accurate schemes that generalize [13] Godunov's scheme, its second-order accurate MUSCL extension ([14,15]), and the total variation diminishing (TVD) scheme [2] to arbitrary order of accuracy. In contrast to the earlier second-order TVD schemes which drop to first order accuracy at local extrema and maintain second-order accuracy in smooth regions, the new ENO schemes are uniformly high order accurate throughout, even at critical points. The ENO schemes use a reconstruction algorithm that is derived from a new interpolation technique that when applied to piecewise smooth data gives high order accuracy whenever the function is smooth but avoids a Gibbs phenomenon at discontinuities. An adaptive stencil of grid points is used; therefore, the resulting schemes are highly nonlinear even in the scalar case. Some schemes constructed in this way were:

Considering ENO schemes, [9] Harten and Osher presented a hierarchy of uniformly high order accurate schemes that generalize [13] scheme, and its second-order accurate extension of monotonic upstream schemes for conservation laws (MUSCL) ([14,15]) and total variation diminishing (TVD) schemes [2,16] to arbitrary order of accuracy.