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A data assimilation approach for non-Newtonian blood flow simulations in 3D geometries

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ABSTRACT

Blood flow simulations can play an important role in medical training and diagnostic predictions associated to several pathologies of the cardiovascular system. The main challenge, at the present stage, is to obtain reliable numerical simulations in the particular districts of the cardiovascular system that we are interested in. Here, we propose a Data Assimilation procedure, in the form of a non linear optimal control problem of Dirichlet type, to reconstruct the blood flow profile from known data, available in certain parts of the computational domain. This method will allow us to obtain the boundary conditions, not fully determined by the physics of the model, in order to recover more accurate simulations. To solve the control problem we propose a Discretize then Optimize (DO) approach, based on a stabilized finite element method. Numerical simulations on 3D geometries are performed to validate this procedure. In particular, we consider some idealized geometries of interest, and real geometries such as a saccular aneurysm and a bypass. We assume blood as an homogeneous fluid with non-Newtonian inelastic shear-thinning behavior. The results show that, even in the presence of noisy data, accuracy can be improved using the optimal control approach.

1. Introduction

The increasing collaboration between scientists working in multidisciplinary areas such as medical researchers and clinicians, mathematicians and bioengineers has contributed to data information exchange that can be used in the numerical simulations providing more realistic results and increasing the advantages in therapy prediction and surgical planning.

The techniques based on the inclusion of data measurements in the numerical simulations are known, in the literature, as Data Assimilation (DA). Such techniques have already been used in other engineering fields like geophysics and meteorology (for an overview, see [40] and the references therein). More recently, DA variational approaches were also used to improve blood flow simulations [18,24]. Besides, techniques based on the unscented Kalman filter were suggested to perform parameter estimation in cardiovascular modeling (see [7,32,34] and [31] for an overview).

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In [18], several approaches were compared, namely the domain splitting method, the matrix updating technique and the variational approach. The later was shown to give the best results among the three techniques. This approach consists in minimizing the misfit between the observed data and the solution of the model, by controlling certain free parameters. The authors assumed the viscosity function in (1) to be constant, and took the value of the pressure on the inlet boundary as the parameter to control. In [24] this approach was reformulated to include the possibility of controlling the velocity profile itself. The authors performed a parameter fitting for the cost function and verified the robustness of the approach, with respect to noise reduction, in a 2D idealized stenosis. In [38], the authors used DA techniques as a tool for improving the accuracy of computational domains reconstructed from medical images. The existence of solution for these mathematical approaches to the DA problem was not established yet. However, numerical solutions based on a Discretized then Optimize (DO) approach was shown to be successful in this frame. The DO consists in first discretizing the variational (optimal control) problem describing the DA method, and then solving the resulting nonlinear finite-dimensional optimization problem. The DO approach is often put side by side with the Optimize then Discretize (OD) approach, but it is not clear yet which is the most suitable in the frame of fluid control. For example, in [8] and [29], it is indicated that, for certain kinds of parabolic problems, a DO approach is preferred. For fluid control problems the authors show that the discrete solution may not converge to the solution of the continuous problem while using OD. Conversely, in [16], it is shown that, in certain cases of stabilized advection equations, the OD has better asymptotic convergence properties. For more details on this issue see, [11,18,25], or [28].

In this paper we propose a DA technique to control the velocity inlet profile in 3D domains. As a first approach, the model is assumed to be stationary, in order to neglect the fluid interaction with the vessel walls. This assumption allows to consider the modeling of some pathologies for which only a long term blood flow profile is required. One of such pathologies is atherosclerosis, one of the deadliest cardiovascular diseases. For such large time scales, it becomes unreasonable to model the short scale pulsatile effects. In future work, we should consider the assimilation of space dependent data obtained at different time steps. Dealing with time dependent data will allow to model clinical problems in which the pulsatile nature of blood is determinant. The methodology adopted includes the use of the Sequential Quadratic Programming method [19] to solve the discretized optimization problem. The result is a large scale finite dimensional, nonlinear optimization problem. Here, we investigate the robustness of this approach in such cases. Therefore, we consider different idealized and realistic geometries. The later represent particular districts of the arterial system, under pathological stages, which were obtained from medical images. We assess the robustness of the method in such domains and analyze the influence of the location of data measurements.

This paper is organized as follows. In Section 2 we introduce the 3D blood flow mathematical model, considered as a shear-thinning non-Newtonian fluid, and we describe the variational approach for the DA problem. In Section 3 we show how to use the DO approach to solve the DA problem and describe the optimization algorithm. In Section 4, we present and discuss the numerical results obtained for different 3D geometries.

2. Data Assimilation approach

To outline the Data Assimilation procedure, we first describe the fluid flow equations as well as the viscosity model, used in the numerical simulations.

In this work we assume that blood is an incompressible, shear-thinning fluid at constant temperature. By fixing Ω as a 3D domain of interest, the equations describing the linear momentum and mass conservation, under the steady assumption, can be written as

$$\begin{cases} -div\,(\tau(D\mathbf{u})) + \rho(\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p = \mathbf{f} & \text{in } \Omega \\ div\,\mathbf{u} = 0 & \text{in } \Omega \end{cases} \tag{1}$$

complemented with suitable boundary and initial conditions. Here $\mathbf{u}$ refers to the velocity field and p to the pressure. The constant parameter ρ represents the density of the fluid and $\mathbf{f}$ the body force. The tensor of viscous stresses is represented by

$$\tau = 2\mu(\dot{\gamma})D\mathbf{u},$$

where μ is the dynamic viscosity, $\dot{\gamma}$ refers to the shear rate

$$\dot{\gamma} = \sqrt{\frac{1}{2}(\nabla\mathbf{u} + (\nabla\mathbf{u})^T) : (\nabla\mathbf{u} + (\nabla\mathbf{u})^T)} = \sqrt{2}|D\mathbf{u}|$$

and D represents the strain rate tensor

$$D\mathbf{u} = \frac{1}{2}\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right).$$

For a shear-thinning fluid we assume the viscosity to decrease with growing shear rate. Therefore, we assume μ to be given by

$$\mu(\dot{\gamma}) = \mu_\infty + (\mu_0 - \mu_\infty)F(\dot{\gamma}), \tag{2}$$