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The relaxation modulus-based matrix splitting iteration method for solving linear complementarity problems of positive definite matrices

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ABSTRACT

In this paper, the relaxation modulus-based matrix splitting iteration method is established for solving the linear complementarity problem of positive definite matrices. The convergence analysis and the strategy of the choice of the parameters are given. Numerical examples show that the proposed method with the new strategy is efficient and accelerates the convergence performance with less iteration steps and CPU times.

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1. Introduction

The linear complementarity problem (LCP(q, A)) consists of finding a vector $z \in \mathbf{R}^n$ such that

$$r = Az + q \geq 0, z \geq 0, \text{ and } z^T r = 0, \tag{1}$$

where $A \in \mathbf{R}^{n\times n}$ and $q \in \mathbf{R}^n$.

The LCP(q, A) is widely used in many applications, e.g., the economies with institutional restrictions upon prices, the linear and quadratic programming, the free boundary problems, the optimal stopping in Markov chain, the network equilibrium problems and the contact problems; see [7,9,15] for details.

In the recent years, some solvers of LCP(q, A) were given based on the modulus iteration method presented by van Bokhoven in [17]. In particular, Bai established the modulus-based matrix splitting iteration method in [2], which was extended to more general cases by Li [12], including the modified modulus method [8] and the nonstationary extrapolated modulus algorithms [11] as its special cases. In addition, Hadjidimos et al. [10] and Zhang [18] proposed scaled extrapolated modulus algorithms and two-step modulus-based matrix splitting iteration methods, respectively. Moreover the modulus-based synchronous multisplitting iteration methods and modulus-based synchronous two-stage multisplitting iteration methods were established in [5] and [6]. In [22] Zheng and Yin proposed a class of accelerated modulus-based matrix splitting iteration methods, which can be generalized in [14]. Li and Zheng presented the preconditioned modulus-based matrix splitting iteration method in [13]. The relaxation modulus-based matrix splitting iteration method was given and the strategy of the relaxation parameters was discussed in [20]. The global convergence conditions of these methods are discussed when the system matrix is either a positive definite matrix or an H_+-matrix; see the references mentioned above

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for details. Moreover, the modulus method has been used also for the solution of least square problems [21] and ill-posed inverse problems [4].

In this paper, we focus on the relaxation modulus-based matrix splitting iteration method presented in [20], and give its theoretical analysis when A is a positive definite matrix, which is not considered in [20]. Numerical examples are given to show that the relaxation modulus-based matrix splitting iteration method is also efficient with the new strategy of choosing parameters for positive definite matrices.

First we introduce some notations and definitions. Let $sp(A)$ and $\rho(A)$ be the spectrum and the spectral radius of a matrix A. For two $m\times n$ real matrices $B=(b_{ij})$ and $C=(c_{ij})$ the order $B\geq C(B>C)$ means $b_{ij}\geq c_{ij}(b_{ij}>c_{ij})$ for all i and j. Let e be an $n\times 1$ vector whose elements are all equal to 1, $A=(a_{ij})\in\mathbf{R}^{n\times n}$ and let $D_A=\mathrm{diag}(A)$ and $B_A=D_A-A$. By $|A|$ we denote $|A|=(|a_{ij}|)$ and the comparison matrix of A is $\langle A\rangle=(\langle a_{ij}\rangle)$, defined by $\langle a_{ij}\rangle=|a_{ij}|$ if $i=j$ and $\langle a_{ij}\rangle=-|a_{ij}|$ if $i\neq j$. The matrix A is called (e.g., see [1]) a Z-matrix if all of its off-diagonal entries are non-positive, an M-matrix if it is a Z-matrix with $A^{-1}\geq 0$, and an H-matrix if its comparison matrix $\langle A\rangle$ is an M-matrix. Specially, an H-matrix with positive diagonal entries is called an H_+-matrix (e.g., see [3]). The splitting $A=M-N$ is called an H-splitting if $\langle M\rangle-|N|$ is an M-matrix (e.g., see [19]). Let $||\cdot||_v$ be an arbitrary vector norm on $\mathbf{R}^n$ and $Q\in\mathbf{R}^{n\times n}$ be a arbitrary nonsingular matrix. Then for $\forall x\in\mathbf{R}^n$ and $\forall X\in\mathbf{R}^{n\times n}$, $||x||_{Q,v}=||Qx||_v$ is a vector norm, while $||X||_{Q,v}=||QXQ^{-1}||_v$ is a matrix norm; see [16].

The rest of this paper is organized as follows. In Section 2, we review the relaxation modulus-based matrix splitting iteration method for solving LCP(q, A). In Section 3, we give the convergence analysis when A is a positive definite matrix and discuss the choice of the relaxation parameter. Numerical examples are given in Section 4 and a conclusion remark is given in the final section.

2. The existing methods

It is well known that solving LCP(q, A) is equivalent to solving the modulus equation [17]:

$$(A+I)x+(A-I)|x|-q=0, \tag{2}$$

where q and A are given by (1). If x is a solution of (2), $z=x+|x|$ is a solution of LCP(q, A).

By introducing the splitting technique, Bai presented the modulus-based matrix splitting iteration method:

Method 2.1 ([2]). Let q and A be given by (1) and $A=M-N$ be a splitting of A. Given an initial vector $x^{(0)}\in\mathbf{R}^n$, for $k=1,2,\ldots$ until the iteration sequence $\{z^{(k)}\}_{k=1}^{+\infty}\subset\mathbf{R}^n$ converges, compute $x^{(k)}\in\mathbf{R}^n$ by solving the linear system

$$(\Omega+M)x^{(k)}=Nx^{(k-1)}+(\Omega-A)|x^{(k-1)}|-\gamma q, \tag{3}$$

and set

$$z^{(k)}=\frac{1}{\gamma}(|x^{(k)}|+x^{(k)}).$$

Here Ω is an $n\times n$ positive diagonal matrix and γ is a positive constant.

By Method 2.1, one may deduce a series of modulus-based matrix splitting iteration methods by different choices of the splitting; see [2] for details.

Let Ω_1, Ω_2 be positive diagonal matrices and $A\Omega_1=M_{\Omega_1}-N_{\Omega_1}$. Method 2.1 was generalized in [12], where the iteration is given by

$$(\Omega_2+M_{\Omega_1})x^{(k)}=N_{\Omega_1}x^{(k-1)}+(\Omega_2-A\Omega_1)|x^{(k-1)}|-q, \tag{4}$$

$$z^{(k)}=\Omega_1(|x^{(k)}|+x^{(k)}).$$

By introducing a nonsingular parameter matrix P to mix the new approach vector and the old approach vector before the next iteration, Zheng et al. [20] further generalized the iteration (4) and obtained the following relaxation modulus-based matrix splitting iteration method for LCP(q, A).

Method 2.2 ([20]). For any given positive diagonal matrices Ω_1 and Ω_2, let $A\Omega_1=M_{\Omega_1}-N_{\Omega_1}$ be a splitting of the matrix $A\Omega_1\in\mathbf{R}^{n\times n}$. Given an initial vector $x^{(0)}\in\mathbf{R}^n$, for $k=1,2,\ldots$ until the iteration sequence $\{z^{(k)}\}_{k=1}^{+\infty}\subset\mathbf{R}^n$ converges, compute $x^{(k)}\in\mathbf{R}^n$ by solving the linear systems

$$\begin{cases}(\Omega_2+M_{\Omega_1})x^{(k-\frac{1}{2})}=N_{\Omega_1}x^{(k-1)}+(\Omega_2-A\Omega_1)|x^{(k-1)}|-q,\\ x^{(k)}=(I-P^{(k)})x^{(k-1)}+P^{(k)}x^{(k-\frac{1}{2})}\end{cases} \tag{5}$$

and set

$$z^{(k)}=\Omega_1(|x^{(k)}|+x^{(k)}).$$

It follows from (5) that

$$x^{(k)}=(I-P^{(k)})x^{(k-1)}+P^{(k)}(\Omega_2+M_{\Omega_1})^{-1}[N_{\Omega_1}x^{(k-1)}+(\Omega_2-A\Omega_1)|x^{(k-1)}|-q], \tag{6}$$

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