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# Analytical solution of MHD slip flow past a constant wedge within a porous medium using DTM-Padé

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ABSTRACT

The objective of present study is to investigate the two-dimensional magnetohydrodynamic (MHD) flow of a viscous fluid over a constant wedge immersed in a porous medium with velocity slip condition. The flow is induced by suction/injection and also by the main-stream flow that is assumed to vary in a power-law manner with co-ordinate distance along the boundary. Similarity transformations are used to convert the governing nonlinear boundary layer equations into a third order Falkner–Skan equation. This equation is solved analytically by using a novel analytical method called DTM-Padé technique which is a combination of the differential transformation method and the Padé approximation. This method is applied to give solutions of equation with boundary condition at infinity. Graphical results are presented to investigate the effects of the velocity slip parameter, Hartmann number, permeability, suction/injection parameter and nonlinear pressure gradient on the flow-field. Further, the results of the present analysis have been compared with the corresponding results available in literature. Our results have been found in excellent agreement.



## 1. Introduction

The steady laminar flow past a fixed wedge was first analyzed by Falkner and Skan [1] to illustrate the application of Prandtl's boundary layer theory. They proposed a similarity transformation that can be utilized to reduce the nonlinear boundary layer equations to a nonlinear third order ordinary differential equation, which is well known as Falkner–Skan equation. A large amount of literature on this subject is available in Schlichting and Gersten [2], Leal [3] and Ishak et al. [4].

The study of MHD boundary layer flow governed by Falkner–Skan type equation has received considerable attention due to its important engineering applications in devices such as power generator, the cooling of reactor, the design of heat exchangers, MHD accelerators, polymer industry, and spinning of filaments. On account of this reason only, Andersson [5] studied the effect of viscoelastic fluid past a stretching sheet in the presence of magnetic field. Raptis et al. [6] discussed the effect of thermal radiation on MHD flow. The two-dimensional magnetohydrodynamic (MHD) stagnation-point flow of an incompressible micropolar fluid over a nonlinear stretching surface is studied by Hayat et al. [7]. Rashidi and Erfani [8] analyzed the similarity solution for the MHD Hiemenz flow against a flat plate with variable wall temperature in a

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porous medium. Kudenatti et al. [9] demonstrated the two-dimensional magnetohydrodynamic flow of a viscous fluid over a constant wedge immersed in a porous medium.

All the above mentioned studies continued their discussions by assuming the no-slip boundary conditions. The no-slip boundary condition, in which it is assumed that a liquid adheres to a solid boundary, is one of the central tenets of the Navier–Stokes theory. However, there are situations where-in this condition does not hold. Partial velocity slip may occur on the stretching boundary when the fluid is particulate such as emulsions, suspensions, foams and the polymer solutions. The non-adherence of the fluid to a solid boundary, also known as velocity slip, is a phenomenon that has been observed under certain circumstances [10]. Recently, many researchers [11–13], etc. investigated the flow problems taking slip flow condition at the boundary. The fluids that exhibit boundary slip flow have important technological applications such as in the polishing of artificial heart valves and internal cavities. Since some coated surfaces such as Teflon resist adhesion, the no-slip condition is replaced by Navier's partial slip condition, where the slip velocity is proportional to the local shear stress. However, experiments suggest that the slip velocity also depends on the normal stress. A number of models have been advanced for describing the slip that occurs at solid boundaries. A new dimension is added to the above mentioned studies by considering the effects of partial slip at the stretching wall. The study of the magnetohydrodynamic boundary layer flow of a conducting fluid is also important as it finds applications in variety of stretching sheet problems. Representative studies dealing with such effects can be found in Turkyilmazoglu [14–23].

With the advent of numerous symbolic software such as MATHEMATICA, MAPLE, MATLAB and so on in recent years, much attention has been paid by researchers to the newly developed methods to construct approximate analytic solutions of nonlinear equations. In this context, methods like homotopy perturbation method [24,25], homotopy analysis method (HAM) [26–31], variational iteration method [32,33] and the differential transform method (DTM) [34] are worthwhile mentioning. DTM is a semi exact method which does not require the presence of small parameters in the equation for its application. One of the semi-exact methods which does not need small parameters is the DTM. The DTM was first introduced by Zhou [34], who solved the electric circuit analysis problems containing linear and nonlinear differential equations. DTM builds on an iterative procedure for an analytic solution of ordinary and partial differential equations in the way of polynomial. Hence it differs from the famous Taylor series method, which needs large mathematical computation. The benefits of using DTM are that it can reduce the amount of calculation as compared to that of the Taylor series method and also it can be applicable to nonlinear differential equations without the help of discretization, linearization and perturbation, and therefore it is not affected by errors associated with discretization. Consequently, Chen and Ho [35] extended this scheme for solving partial differential equations. Ayaz [36] used it to investigate the system of differential equations. Arikoglu and Ozkol [37] applied it to difference equations. Darania and Ebadian [38] applied it to integro-differential equations. In recent years, the DTM has been applied successfully to solve many types of problems such as the linear partial differential equations of fractional order [39], nonlinear oscillatory systems [40], multi-order fractional differential equations [41], hyperchaotic Rössler system [42], fourth order boundary value problems [43], Volterra integral equation with separable kernels [44], a new algorithm for calculating two-dimensional differential transform of nonlinear functions [45], linear and nonlinear Schrödinger equations [46], free vibration analysis of circular plates [47], solving fuzzy differential equations [48]. All these successful applications of DTM verify its validity, effectiveness and flexibility. Though the results obtained by DTM in case of the differential equations on unbounded domains are valid in small region, but in a wider range they are invalid. It happens due to the divergent nature of the closed series solution investigated by DTM when the independent variable of the problem tends to infinity. Boyd [49] and other researchers have proved that the power series in unbounded domain is not useful for handling boundary value problems. But the DTM-Padé technique has been adopted and used recently by Su and Zheng [50] to overcome such a difficulty and to get approximate solutions for a wide range.

So, the motivation behind this paper is to extend DTM with Padé approximant for solving MHD boundary layer flow over a constant wedge. The investigation has been made to study the effect of slip condition and comparisons had been made of the results obtained by DTM-Padé with published results by Kudenatti et al. [9]. The graphical results for velocity slip parameter $L$ show an increase in dimensionless velocity for increasing values of $L$ in both the cases of suction and injection.

## 2. Problem statement and mathematical formulation

Let us consider the MHD two-dimensional flow of a viscous, incompressible and electrically conducting fluid over a constant wedge through porous medium. Here $x$-axis is directed towards the wedge surface and $y$-axis normal to the flow (see Fig. 1). The fluid flow is confined only to the half-space $y > 0$. In the flow process, the Reynolds number is assumed to be large. As a consequence of this, the entire flow-field is divided into two regions. One region is confined near to the wedge surface where the viscosity plays a dominant role. The second region is away from the wedge surface where zero shear viscosity is important. The wedge is immersed inside a porous medium and a magnetic field of $B(x)$ is applied to it in a direction normal to the flow. On account of the interaction of the electromagnetic field inside the porous medium, a velocity field is developed. Let $\vec{q} = (u, v)$ be the velocity vector, where $u$ and $v$ are velocity components in the direction s of $x$- and $y$-axes, respectively.

Thus, the MHD flow equations under these assumptions are (cf. [9]):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$