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Finite-time stability and stabilization for Itô-type stochastic Markovian jump systems with generally uncertain transition rates[☆]

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ABSTRACT

This paper investigates the problems of finite-time stability and stabilization for stochastic Markovian jump systems with generally uncertain transition rates (SMJSwGUTRs). Firstly, a less conservative stability criterion is presented by a mode-dependent approach. Then, the state feedback controller and observer-based controller are designed by a cone complementary linearization method, and two specific algorithms are presented. Finally, an example is used to illustrate our results.

1. Introduction

Recently, Markovian jump systems (MJSs) have attracted much attention by virtue of modeling the abrupt phenomenon such as random failures, sudden environment changes, see [1–7]. And also, a wealthy of problems have been studied, such as the sliding control [8,9], robust H_∞ control [10], fault detection [11]. However, in most existing results, the transition probabilities are always assumed to be completely known [12,13] or partially known [14], that means, at least a part of elements in the transition rate matrix need to be accurately measured. In fact, it is difficult to achieve the accurate transition rate in many actual applications due to the short of technology and expensive cost. Thus, the model of MJSwGUTRs has been paid more attention. In concrete terms, GUTRs means that the transition probabilities can be fully unknown or only its estimate is known. Some significant results have been reported. For example, Kao et al. [15] studied the delay-dependent stability for MJSwGUTRs. In [16], the problem of robust H_2 control for Markovian jump linear systems with uncertain transition probabilities was investigated. Some other related papers can be found in [17–19] and the references therein.

In the meanwhile, the finite-time stability has been widely studied in many practical fields, such as aircraft systems, economic-controlled systems, see [20–23]. Roughly speaking, if the state of dynamic systems does not exceed a certain threshold during a fixed time interval, then the system is said to be finite-time stable [24]. As a matter of fact, this kind of stability is more concerned about the transient performance of systems within a short time. A great deal of results on the finite-time stability and stabilization have been reported. For instance, Yan et al. [25] investigated the finite-time stability and stabilization for the Itô stochastic systems with Markovian switching in terms of a mode-dependent parameter

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approach. Chen et al. [26] studied finite-time stability for switched stochastic delayed systems, and obtained some mean-square finite-time stability conditions. Readers can refer to [27,28] for more results on finite-time stability.

Although there have been many results on the finite-time stability and stabilization, there are nearly no results on this kind of problems of the model of Itô stochastic Markovian jump systems with GUTRs. In the existing literatures, such as, [29,30], an important method is to use the following inequality

$$\frac{d[x'(t)P_ix(t)]}{dt} < \beta[x'(t)P_ix(t)], \tag{1}$$

where P_i is mode-dependent but the β is mode-independent. Because the β is common to all modes (β is common to P_i), some results obtained are conservative. If β_i is also mode-dependent, that is, one β_i is corresponding to P_i, then some loose conditions will be obtained. Besides, there is nearly no literatures to investigate finite-time stability problems of Itô-type SMJSwGUTRs.

Motivated by the above discussions, the problems of finite-time stability and stabilization for Itô-type SMJSwGUTRs are considered in this paper. Because of the special structure of the considered system, the stability analysis and controller design are more difficult than those in existing literatures. The main contributions of this paper are as follows: (1) The system model addressed include generally uncertain transition rates and stochastic noise, which is more complex than that in [29]. (2) Some less conservative conditions for the existence of two kinds of controllers (state feedback and observer-based controllers) are given by a mode-dependent parameter approach. (3) Two new algorithms are given to solve the inequalities arising from the design of state-feedback controllers and observer-based controller.

The rest of this paper is organized as follows. Section 2 gives some preliminaries, definitions and lemmas. In Section 3, we provide a finite-time stability criteria for Itô SMJSwGUTRs. Section 4 gives the design method of state feedback controller and observer-based controller. In Section 5, two algorithms are given. An illustrative example is discussed in Section 6. The last section provides our conclusions.

Notions: X' is the transpose of a matrix X. $X > 0$ means that X is real symmetric and positive definite. $\mathbb{E}[X]$ stands for the expectation of a stochastic variable X. $tr(X)$ is the trace of a matrix X. $\lambda_{max}(X)$ and $\lambda_{min}(X)$ represent the maximum and minimum eigenvalue of a matrix X, respectively. $I_{n\times n}$ stands for $n \times n$ identity matrix. In symmetric block matrices, the asterisk '*' is the term that can be determined by symmetry. diag$\{\cdots\}$ stands for a block-diagonal matrix. The 'wrt' stands for 'with respect to'.

2. Systems model and problems formulation

Consider a kind of stochastic systems with Markovian jumps

$$\begin{cases} dx(t) = [A_0(\theta_t)x(t) + B_0(\theta_t)u(t)]dt + [A_1(\theta_t)x(t) + B_1(\theta_t)u(t)]dw(t), \\ y(t) = C(\theta_t)x(t), \;\; x(0) = x_0 \in R^n, \;\; \theta(0) = \theta_0, \end{cases} \tag{2}$$

where $x(t) \in \mathcal{R}^n$ is system state, $u(t) \in \mathcal{R}^m$ is control input, and $y(t) \in \mathcal{R}^p$ is measurement output; $w(t)$ is an one-dimensional Winner process; θ_t is a right-continuous Markov chain and takes values in $S = \{1, 2, \dots, N\}$ with the transition probabilities

$$P = \{\theta_{t+\triangle t} = b/\theta_t = a\} = \begin{cases} \gamma_{ab}\triangle t + o(\triangle t), & a \neq b, \\ 1 + \gamma_{aa}\triangle t + o(\triangle t), & a = b, \end{cases}$$

where $\triangle t > 0$, $\gamma_{ab} \geq 0$ is the transition rate from mode a to mode b in the time interval $\triangle t$, which satisfies $\gamma_{ab} > 0$, $a \neq b$ and $\gamma_{aa} = -\sum_{a\neq b} \gamma_{ab}$. For $\theta_t = a$, $A_0(\theta_t)$, $A_1(\theta_t)$, $B_0(\theta_t)$, $B_1(\theta_t)$ and $C(\theta_t)$ are known matrices and are denoted by A_{0a}, A_{1a}, B_{0a}, B_{1a} and C_a for simplicity. θ_t and $w(t)$ are independent.

Considering the uncertain transition rates in jumping process, the elements in the matrix Γ can be unknown, or only its lower and upper bounds are known. For example, the transition rate matrix of system (2) with four jumping modes can be expressed as

$$\Gamma = \begin{bmatrix} \gamma_{11} + \triangle\gamma_{11} & ? & \gamma_{13} + \triangle\gamma_{13} & ? \\ ? & \gamma_{22} + \triangle\gamma_{22} & ? & ? \\ ? & ? & ? & \gamma_{34} + \triangle\gamma_{34} \\ ? & \gamma_{42} + \triangle\gamma_{42} & ? & \gamma_{44} + \triangle\gamma_{44} \end{bmatrix}, \tag{3}$$

where $\hat{\gamma}_{ab} = \gamma_{ab} + \triangle\gamma_{ab}$, γ_{ab} and $\triangle\gamma_{ab} \in [-\mu_{ab}, \; \mu_{ab}]$ ($\mu_{ab} \geq 0$) represent the estimate value and estimate error of the uncertain transition rate $\hat{\gamma}_{ab}$, respectively, and they satisfy $\gamma_{aa} = -\sum_{b=1,\; b\neq a}^{N} \gamma_{ab}$, $\triangle\gamma_{aa} = -\sum_{b=1,\; b\neq a}^{N} \triangle\gamma_{ab}$, $|\gamma_{ab}| > |\mu_{ab}|$. γ_{ab} and μ_{ab} are known. The notation "?" represents the completely unknown elements. For $a \in S$, we denote $\mathrm{U^a} = \mathrm{S_k^a} + \mathrm{S_{uk}^a}$, with

$$\mathrm{S_k^a} \triangleq \{\mathrm{b} : \text{the estimate value of } \hat{\gamma}_{\mathrm{ab}} \text{ is known for } \mathrm{b} \in \mathrm{S}\},$$
$$\mathrm{S_{uk}^a} \triangleq \{\mathrm{b} : \text{the estimate value of } \hat{\gamma}_{\mathrm{ab}} \text{ is unknown for } \mathrm{b} \in \mathrm{S}\}.$$

Moreover, if $\mathrm{S_k^a} \neq \varnothing$, it is further described as

$$\mathrm{S_k^a} = \{k_1^a, k_2^a, \dots, k_m^a\}, \;\; 1 \leq m \leq N,$$