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Exponential stability for a class of memristive neural networks with mixed time-varying delays

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ABSTRACT

A new general hybrid neural networks with inertial term and mixed time-varying delays are proposed here by using the memristors connections. Then by building appropriate Lyapunov functionals and inequality technique, some new conditions assuring the global exponential stability of the hybrid neural networks are derived. The circuit implementation of the proposed hybrid neural networks are also presented here. In addition, the new proposed results here enrich and extend the earlier publications on neural networks. Lastly, numerical simulations show the effectiveness of our results.

1. Introduction

Mauro and Conti et al. [1] shown that quasi-active membrane behavior of neurons can be modeled by adding inductance which makes the membrane to have electrical tuning, filtering behaviors. It also has been proved that with an inductance in semicircular canals of certain animals, membrane of a hair cell can be realized by equivalent circuits [2]. The charge or flux q of an electron element with inertial can be inertial with the tendency to be unchanged, this fact have been proved in [3]. Therefore, bringing an inertial term into a neural system exists evident engineering and biological backgrounds. The authors in [4] proved that when the inertial item is introduced into neural networks, the dynamical behaviors would be more complex. With the development and application of inertial neural networks, the studies of such nonlinear system are necessary and meaningful of both theoretical and potentially practical application, and these years pay many researchers' attention, e.g., see [5–7]. Compared to conventional neural networks with first order derivative of states, inertial neural networks are second order derivative of states, little attention has been given to the inertial neural networks.

It is well-known that the stability of neural networks has historically assumed a position of great importance in system theory and has been studied extensively in recent years, e.g., see [8–12]. On the other hand, during the implementation of neural networks, time delays in particular time-varying delays are unavoidably encountered in the signal transmission among the neurons due to the finite switching speed of neurons and amplifiers, which will affect the stability of the neural system [13,14]. Meanwhile, neural networks often have a spatial extent because of the presence of an amount of parallel pathways of varying of axon size and lengths. Then, there may exists either a distribution of conduction velocities along these pathways or a distribution of propagation delays over a period of time in some cases, which may cause another type of time delays, namely, distributed time delays in neural networks. And these years appeared many works, e.g., see [15–20].

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It is have been showed that memristor devices have many promising applications, one of which is to emulate synaptic behaviors [21,22]. So, we can replace resistors with memristors in the conventional circuit implementation of neural network to design a new model called memristive neural networks to emulate the human brain. Memristive neural networks are a class of state-dependent nonlinear systems from a systems-theoretic point of view [23–32]. Such system family can reveal jumped, transient chaos of rich and complex nonlinear behaviors. In order to allow the memristors to be readily used in emerging technologies, the stability of such state-dependent nonlinear system should be studied in the first position, therefore, it is of both theoretical and practical importance to study the problem of stability for memristive neural networks.

And for memristive neural networks, there are also some works for stability, e.g., Wu and Zeng [28] investigated the stability of delayed memristive neural networks; Zhang and Shen [29] discussed the exponential stabilization of chaotics memristive neural networks via intermittent control.

The memristive neural networks with inertial term and mixed time-varying delays is also a system family, however, on the stability of such system, few results are found in the existing literatures. So, it is of great importance to fill this gap. Motivated by the above discussions, in this paper, we will derive several criteria ensuring exponential stability of memristive inertial neural networks with mixed time-varying delays. The main contribution of the paper lies in the following aspects.

(1) Compared with the results on exponential stability of conventional neural networks [11,17,18], we discuss the global exponential stability of neural networks with memristors.
(2) The circuit implementation of memristive inertial neural networks with mixed time-varying delays is given out.
(3) The stability analysis is extended to the memristive neural networks with inertial term and mixed time-varying delays, this kind of systems are second order derivative of states, which extend the earlier publications.

The organization of this paper is as follows. In Section 2, some preliminaries are introduced. In Section 3, some algebraic conditions concerning global exponential stability are derived. Numerical simulations are given in Section 4. Finally, some conclusions are drawn.

2. Preliminaries

Let $\mathbb{R}^n$ be the space of n-dimensional real column vectors. For any $\varrho=(\varrho_1,\varrho_2,\ldots,\varrho_n)^T\in\mathbb{R}^n$, $\|\varrho\|=\sum_{i=1}^{n}|\varrho_i|$. co$[\gamma^*,\ \gamma^{**}]$ denotes convex hull of $\{\gamma^*,\ \gamma^{**}\}$. $A_{ij}=\max\{|a^*_{ij}|,|a^{**}_{ij}|\}$, $B_{ij}=\max\{|b^*_{ij}|,|b^{**}_{ij}|\}$, $C_{ij}=\max\{|c^*_{ij}|,|c^{**}_{ij}|\}$, $\overline{d}_i=\max\{d^*_i,\ d^{**}_i\}$, $\underline{d}_i=\min\{d^*_i,\ d^{**}_i\}$, $\overline{a}_{ij}=\max\{a^*_{ij},\ a^{**}_{ij}\}$, $\underline{a}_{ij}=\min\{a^*_{ij},\ a^{**}_{ij}\}$, $\overline{b}_{ij}=\max\{b^*_{ij},\ b^{**}_{ij}\}$, $\underline{b}_{ij}=\min\{b^*_{ij},\ b^{**}_{ij}\}$, $\overline{c}_{ij}=\max\{c^*_{ij},\ c^{**}_{ij}\}$, $\underline{c}_{ij}=\min\{c^*_{ij},\ c^{**}_{ij}\}$, $\underline{\mu}_i=\underline{d}_i-\lambda_i$, $\overline{\mu}_i=\overline{d}_i-\lambda_i$, $\nu^+_i=\max\{|\underline{\mu}_i\lambda_i-\alpha_i|,|\overline{\mu}_i\lambda_i-\alpha_i|\}$. $\mathcal{N}=\{1,2,\ldots,n\}$. In this paper, solutions of all systems considered in the following are in Filippov's sense [33].

Based on Kirchhoffs current law and from the circuit of memristive inertial neural networks as shown in Fig. 1, we can get the following equations of the ith subsystem:

$$\begin{aligned}\mathbf{L}_i\mathbf{C}_i\frac{\mathrm{d}^2z_i(t)}{\mathrm{d}t^2}=&-z_i(t)-\mathbf{L}_i\left[\sum_{j=1}^{n}\left(\frac{1}{\mathbf{M}_{ij}}+\frac{1}{\mathbf{M}^*_{ij}}+\frac{1}{\mathbf{M}^{**}_{ij}}\right)\delta_{ij}+\frac{1}{\mathbf{R}_i}\right]\frac{\mathrm{d}z_i(t)}{\mathrm{d}t}\\&+\sum_{j=1}^{n}\frac{\delta_{ij}}{\mathbf{M}_{ij}}f_j(\mathbf{L}_i\dot{z}_j(t))+\sum_{j=1}^{n}\frac{\delta_{ij}}{\mathbf{M}^*_{ij}}f_j(\mathbf{L}_i\dot{z}_j(t-\tau_j(t)))\\&+\sum_{j=1}^{n}\frac{\delta_{ij}}{\mathbf{M}^{**}_{ij}}\int_{t-\rho_j(t)}^{t}f_j(\mathbf{L}_i\dot{z}_j(s))\mathrm{d}s+l_i(t)\ \ t\ge 0,i\in\mathcal{N},\end{aligned}$$

where $\mathbf{L}_i$ is inductance, $z_i(t)$ is current of the inductor, $\mathbf{R}_i$ and $\mathbf{C}_i$ are the resistor and capacitor, respectively. $\mathbf{M}_{ij}$ represents the memristor between the neuron activation function $f_j(\mathbf{L}_i\dot{z}_j(t))$ and $z_i(t)$, $\mathbf{M}^*_{ij}$ represents the memristor between the neuron activation function $f_j(\mathbf{L}_i\dot{z}_j(t-\tau_j(t)))$ and $z_i(t)$, $\mathbf{M}^{**}_{ij}$ represents the memristor between the neuron activation function $\int_{t-\rho_j(t)}^{t}f_j(\mathbf{L}_i\dot{z}_j(s))\mathrm{d}s$ and $z_i(t)$. Here $\delta_{ij}=1$, if $i\neq j$ holds, otherwise, -1. And $l_i(t)$ is the external input, $\tau_j(t)$ and $\rho_j(t)$ corresponds to the transmission delays and satisfies $0\le\tau_j(t)\le\tau$, $\dot{\tau}_j(t)\le\tau_0<1$, $0\le\rho_j(t)\le\rho$, $\dot{\rho}_j(t)\le\rho_0<1$.

In this paper, for simplicity, we let $x_i(t)=\mathbf{L}_iz_i(t)$ and $l_i(t)=0$, from the above equations, then, we have

$$\begin{aligned}\frac{\mathrm{d}^2x_i(t)}{\mathrm{d}t^2}=&-\alpha_ix_i(t)-d_i(\dot{x}_i(t))\frac{\mathrm{d}x_i(t)}{\mathrm{d}t}+\sum_{j=1}^{n}a_{ij}(\dot{x}_i(t))f_j(\dot{x}_j(t))+\sum_{j=1}^{n}b_{ij}(\dot{x}_i(t))f_j(\dot{x}_j(t-\tau_j(t)))+\sum_{j=1}^{n}c_{ij}(\dot{x}_i(t))\\&\times\int_{t-\rho_j(t)}^{t}f_j(\dot{x}_j(s))\mathrm{d}s,\ t\ge 0,i\in\mathcal{N},\end{aligned}\tag{1}$$