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Polynomial stability for wave equations with acoustic boundary conditions and boundary memory damping☆

Chan Li[a], Jin Liang[b], Ti-Jun Xiao[c,*]

[a] School of Sciences, Hangzhou Dianzi University, Zhejiang, 310018, China
[b] Department of Applied Mathematics, Shanghai Jiao Tong University, Shanghai, 200240, China
[c] Shanghai Key Laboratory for Contemporary Applied Mathematics, School of Mathematical Sciences, Fudan University, Shanghai, 200433, China

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ABSTRACT

We study wave equations with acoustic boundary conditions, where only one memory damping acts on the acoustic boundary. Under some conditions on the memory kernel, polynomial energy decay rates are established by using higher-order energy estimates among some other techniques.

1. Introduction

In this paper, we investigate the stability of solutions of the following linear wave equation subject to acoustic boundary condition on one part of the boundary:

$$\begin{cases} u_{tt} - \Delta u = 0, & \text{in } \Omega \times (0,T), \\ u = 0, & \text{on } \Gamma_0 \times (0,T), \\ u_t - \displaystyle\int_0^t g(t-s)z(s)ds + z + z_{tt} = 0, & \text{on } \Gamma_1 \times (0,T), \\ \partial_\nu u = z_t, & \text{on } \Gamma_1 \times (0,T), \\ u(0) = u_0, u_t(0) = u_1, & \text{in } \Omega, \\ z(0) = z_0, z_t(0) = z_1, & \text{on } \Gamma_1. \end{cases} \tag{1.1}$$

Here Ω is a bounded domain in $\mathbb{R}^n$ with smooth boundary $\Gamma = \Gamma_0 \cup \Gamma_1$, Γ_0, Γ_1 being closed, nonempty and disjoint subsets of Γ; ν is outer-normal vector at $x \in \Gamma$; The function u represents the velocity potential of the fluid, and $z(t, x)$ the normal

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* Corresponding author.
E-mail addresses: tjxiao@fudan.edu.cn, xiaotj@ustc.edu.cn (T.-J. Xiao).

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displacement of the point $x \in \Gamma_1$ at time t; $\partial_\nu u = z_t$ means that the outward normal velocity at $x \in \Gamma_1$ equals the boundary motion velocity; $-\int_0^t g(t-s)z(s)ds$ will produce a damping effect and make the system stable. The system models sound wave propagation in a domain with a portion of boundary made of viscoelastic material.

Acoustic model was proposed by Morse and Ingard [12] in order to fully explain the propagation of sound waves in a concert hall, but in a nonstandard way. Beale and Rosencrans [5] improved it in a rigorous mathematical context, and Beale [3,4] analysed the model in both bounded and exterior domains, where it is assumed that each boundary point acts as a spring and does not influence each other. The study of various systems with acoustic boundary conditions has attracted a lot of interest (cf., e.g., [1,2,6–10,13–16] and references therein).

For linear wave equations with the acoustic boundary condition:

$$u_t - \int_0^t g(t-s)z_t(s)ds + z = 0, \quad \partial_\nu u = z_t, \quad \text{on } \Gamma_1 \times (0,T),$$

it is shown in [10] that the associated operator matrix generates a strongly continuous semigroup of contractions on a Hilbert space, and the semigroup is strongly stable [10] (without giving an energy decay rate). In the present paper, we are devoted to establishing energy decay rates for system (1.1), and will prove that the energy is polynomially stable under suitable conditions on the memory kernel g (Theorem 3.1). To our knowledge, there has been no work about the decay rates of acoustic wave energies when only one memory damping acting on the acoustic boundary is employed to stabilize the whole system. Uniform energy decay rates were studied in [6,15] for acoustic wave systems with both internal and boundary memory dampings (concerning the velocity potential u, rather than the normal displacements z of boundary points). Also, polynomial stability was proved in [14] for (1.1) with the memory damping $-\int_0^t g(t-s)z(s)ds$ replaced by a frictional damping z_t. In the case of memory case, the kernel g will affect energy decay rates, and the handling of the system is quite different from the case of frictional damping.

2. Preliminaries

The following assumptions and notations will be used throughout the paper.

Assumptions. (A-1) the geometric condition: there exists $x_0 \in \mathbb{R}^n$ and a positive constant δ such that

$$(x-x_0)\cdot\nu(x) \le 0, \quad x \in \Gamma_0, \qquad (x-x_0)\cdot\nu \ge \delta > 0, \quad x \in \Gamma_1.$$

A typical example is

$$\Omega = \{x : 1 < |x| < 2\},\ \Gamma_0 = \partial B_1(0),\ \Gamma_1 = \partial B_2(0),\ x_0 = \mathbf{0}.$$

(A-2) $g \in C^5(\mathbb{R}^+ \to \mathbb{R}^+)$ is a decreasing function satisfying

$$g(0) > 0, \quad l := \int_0^\infty g(s)ds < 1,$$

$$\text{meas}\{s \ge 0;\ g'(s) = 0\} = 0, \tag{2.1}$$

and

$$|g^{(i)}(s)| \le c_0 g(s) \quad (i = 1, \dots, 4), \qquad |g'''(s)| + |g^{(5)}(s)| \le -c_1 g'(s) \quad \text{for } s > 0.$$

Typical examples are

$$g(s) = le^{-s}, \quad \text{or} \quad g(s) = \frac{l\alpha}{(1+s)^{\alpha+1}}, \ (\alpha > 0)$$

satisfying the assumption (A-2).

Notations: By $\langle\cdot,\cdot\rangle_\Omega$ we denote the inner product on space $L^2(\Omega)$, $\langle\cdot,\cdot\rangle_{\Gamma_1}$ the inner product on space $L^2(\Gamma_1)$, $||\cdot||_\Omega$ the $L^2(\Omega)$ norm, and $||\cdot||_{\Gamma_1}$ the $L^2(\Gamma_1)$ norm;

$$R = \max_{x\in\bar{\Omega}} |x - x_0|; \quad H^1_{\Gamma_0}(\Omega) = \{u \in H^1(\Omega);\ u|_{\Gamma_0} = 0\}.$$

Write

$$H(t) = \int_0^t g(s)ds, \quad K_\sigma(s) = \frac{-g'(s)}{g(s)} + \sigma, \quad G_\sigma = \int_0^\infty \frac{g(s)}{K_\sigma(s)}ds$$

for $\sigma \in (0, 1)$. More information about the auxiliary functions $H(t)$, K_σ, G_σ can be found in [11]. We will use c, C to denote generic positive constants.

Wellposedness:

By the semigroup theory, it is not difficult to obtain the existence and uniqueness theorem.