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A class of nonlinear non-instantaneous impulsive differential equations involving parameters and fractional order[☆]

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ABSTRACT

In this article, we study asymptotic and smooth properties of solutions to nonlinear non-instantaneous impulsive differential equations involving parameters of integer order and fractional order. We introduce the concept of continuous dependence and differentiability of solutions and establish sufficient conditions to guarantee the solution depends continuously and is differentiable on the initial condition, impulsive parameters and junction parameters. Finally, two models of non-instantaneous impulsive logistic equations are given to illustrate our results.

1. Introduction

Impulsive differential equations are widely used in applied mathematics [1] and many authors made contributions in mechanics, engineering and population ecology [2–4]. Two main types of impulsive effects are considered: instantaneous impulsive differential equations (IIDEs) and non-instantaneous impulsive differential equations (NIDEs) [5,6].

Fractional calculus extends the classical integer order calculus to an arbitrary order case and it is a tool used to describe complex movements, irregularities, and memory features. Fractional differential equations reflects the basic properties of the material more accurately, and plays an important role in engineering, physics, finance and signal analysis; we refer the reader [7–17] and the reference therein.

The qualitative theory of IIDEs was studied extensively in the literature; see [18–20] and the references therein. Recently, NIDEs was introduced in [6] for studying the dynamics of evolution processes in pharmacotherapy. There are many recent results in the literature on existence, stability and control theory for integer order and fractional order equations; see for example, [21–42].

In this paper, we study asymptotic and smooth properties of solutions of two models described by integer order NIDEs:

$$\begin{cases} x'(t) = f(t, x(t), \lambda_{s_i}), \quad t \in (s_i, t_{i+1}], \quad i \in \{0\} \cup N, \\ x(t_i^+) = g_i(t_i, x(t_i^-), \lambda_{t_i}), \quad i \in N, \\ x(t) = g_i(t, x(t_i^-), \lambda_{t_i}) + h(t, \lambda_{s_i}), \quad t \in (t_i, s_i], \quad i \in N, \\ x(0) = x_0(\lambda_0); \end{cases} \tag{1}$$

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and fractional order NIDEs:

$$\begin{cases} {}^c\mathbf{D}^{\alpha}_{s_i,t}x(t) = f(t, x(t), \lambda_{s_i}), & t \in (s_i, t_{i+1}], \quad i \in \{0\} \cup N, \quad \alpha \in (0,1), \\ x(t_i^+) = g_i(t_i, x(t_i^-), \lambda_{t_i}), & i \in N, \\ x(t) = g_i(t, x(t_i^-), \lambda_{t_i}) + h(t, \lambda_{s_i}), & t \in (t_i, s_i], \quad i \in N, \\ x(0) = x_0(\lambda_0), \end{cases} \tag{2}$$

where λ_0, λ_{t_i}, λ_{s_i} are real parameters, $i \in N$ and ${}^c\mathbf{D}^{\alpha}_{s_i,t}$ denotes the classical Caputo fractional derivative of order α by changing the lower limit s_i, and t_i acts as an impulsive point [43] and s_i acts as a junction point satisfying $s_i < t_{i+1} \to \infty$ with $t_0 = s_0 = 0$, and λ_{t_i}, λ_{s_i} are denoted as impulsive parameters and junction parameters, respectively. Note $x(t_i^+) = \lim_{\epsilon\to 0^+} x(t_i + \epsilon)$ and $x(t_i^-) = \lim_{\epsilon\to 0^+} x(t_i - \epsilon) := x(t_i)$. Let f: $[0, \infty) \times D \times R \to R^n$, g_i: $[t_i, s_i] \times D \times R \to R^n$ and h: $[t_i, s_i] \times R \to R^n$, $i \in N$ where the domain $D \subset R^n$.

The main objective in this paper is to extend the work in [47] to non-instantaneous impulses and fractional order and present sufficient conditions to guarantee continuous dependence and differentiability of solutions with respect to initial condition, impulsive perturbation and junction perturbation.

The rest of this paper is organized as follows. In Section 2, we introduce the definitions of continuous dependence and differentiability of the solutions for (1) and (2). In Section 3, we present the results for the continuous dependence and differentiability of solutions with respect to initial condition, impulsive parameters and junction parameters. Two examples are given in the final section to illustrate our results.

2. Preliminaries

Let $J = [0, \infty)$ and $C(J, R^n)$ be the space of all continuous functions from J into R^n. We recall the piecewise continuous functions space $PC(J, R^n) := \{x : J \to R^n : x \in C((t_k, t_{k+1}], R^n), \quad k = 0, 1, \ldots$ and there exist $x(t_k^+)$ and $x(t_k^-), k = 1, 2, \ldots$ with $x(t_k^-) = x(t_k)\}$ with the norm $\|x\|_{PC} := \sup\{|x(t)| : t \in J\}$.

From [44, Definition 2.1], one can see the representation of the piecewise continuous solution $x \in PC(J, R^n)$ of problem (1) is given by

$$\begin{aligned} & x(t; \lambda_0) = x_0(\lambda_0) + \int_0^t f(s, x(s; \lambda_0), \lambda_0)ds, \quad t \in [0, t_1], \\ & x(t; \lambda_0, \lambda_{t_1}, \ldots, \lambda_{s_{i-1}}, \lambda_{t_i}) = g_i(t, x(t_i^-; \lambda_0, \lambda_{t_1}, \ldots, \lambda_{t_{i-1}}, \lambda_{s_{i-1}}), \lambda_{t_i}) \\ & \quad + h(t; \lambda_{s_i}), \quad t \in (t_i, s_i], \quad i \in N, \\ & x(t; \lambda_0, \lambda_{t_1}, \ldots, \lambda_{s_{i-1}}, \lambda_{t_i}, \lambda_{s_i}) = g_i(s_i, x(t_i^-; \lambda_0, \lambda_{t_1}, \ldots, \lambda_{t_{i-1}}, \lambda_{s_{i-1}}), \lambda_{t_i}) + h(s_i; \lambda_{s_i}) \\ & \quad + \int_{s_i}^t f(s, x(s; \lambda_0, \lambda_{t_1}, \ldots, \lambda_{t_i}, \lambda_{s_i}), \lambda_{s_i})ds, \quad t \in (s_i, t_{i+1}], \quad i \in N, \end{aligned}$$

and by Yang et al. [42], [45, Section 3] and [46, Section 8] one can see the representation of the solution $x \in PC(J, R^n)$ of problem (2) is given by

$$\begin{aligned} & x(t; \lambda_0) = x_0(\lambda_0) + \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} f(s, x(s; \lambda_0), \lambda_0)ds, \quad t \in [0, t_1], \\ & x(t; \lambda_0, \lambda_{t_1}, \ldots, \lambda_{s_{i-1}}, \lambda_{t_i}) = g_i(t, x(t_i^-; \lambda_0, \lambda_{t_1}, \ldots, \lambda_{t_{i-1}}, \lambda_{s_{i-1}}), \lambda_{t_i}) \\ & \quad + h(t; \lambda_{s_i}), \quad t \in (t_i, s_i], \quad i \in N, \\ & x(t; \lambda_0, \lambda_{t_1}, \ldots, \lambda_{s_{i-1}}, \lambda_{t_i}, \lambda_{s_i}) = g_i(s_i, x(t_i^-; \lambda_0, \lambda_{t_1}, \ldots, \lambda_{t_{i-1}}, \lambda_{s_{i-1}}), \lambda_{t_i}) + h(s_i; \lambda_{s_i}) \\ & \quad + \frac{1}{\Gamma(\alpha)} \int_{s_i}^t (t-s)^{\alpha-1} f(s, x(s; \lambda_0, \lambda_{t_1}, \ldots, \lambda_{t_i}, \lambda_{s_i}), \lambda_{s_i})ds, \\ & t \in (s_i, t_{i+1}], \quad i \in N. \end{aligned}$$

For brevity, we let $x(t;\cdot)$ and $x(t;\cdot^*)$ be the solutions of the original equations and the perturbation equations respectively, where the perturbation is around the parameters.

Next, inspired by Dishlieva [47], we introduce some definitions of continuous dependence and differentiability of the solutions to (1) and (2) with respect to the impulsive perturbations, respectively.

Definition 2.1. The solution of problem (1) depends continuously on initial parameter λ_0, impulsive parameters λ_{t_i}, $i \in N$ and junction parameters λ_{s_i}, $i \in N$, respectively, provided that the following limits

$$\begin{aligned} & \lim_{\lambda_0^* \to \lambda_0} x(t; \lambda_0^*) = x(t; \lambda_0), \quad t \in [0, t_1], \\ & \lim_{\substack{\lambda^*_{s_{i-1}} \to \lambda_{s_{i-1}} \\ \lambda^*_{t_i} \to \lambda_{t_i}}} x(t; \lambda_0, \lambda_{t_1}, \ldots, \lambda^*_{s_{i-1}}, \lambda^*_{t_i}) = x(t; \lambda_0, \lambda_{t_1}, \ldots, \lambda_{s_{i-1}}, \lambda_{t_i}), \quad t \in (t_i, s_i], \quad i \in N, \end{aligned}$$