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Novel inequality with application to improve the stability criterion for dynamical systems with two additive time-varying delays☆

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ABSTRACT

In this paper, the stability of the system with two additive time-varying delay components is studied and improved stability condition is obtained. It firstly establishes two novel integral inequalities, which are better than the same type inequalities found in the literature. Secondly, a new constructed Lyapunov functional is constructed based on the additive time-varying delays property. Following two steps to handle the Lyapunov functional, the delay-dependent stability condition is obtained which in terms of linear matrix inequalities. Finally, two numerical examples are given to verify the effectiveness of the proposed method and the superiority of the results.

1. Introduction

During the last several decades, many kinds of delays are frequently occur in many practical systems, such as economic systems, engineering systems, neural networks, networked control systems. Recently, many articles provided novel approaches to investigate the less conservative stability criteria for delay dynamical systems [1–7], while other references [8,18,25] and the references therein provided different methods to study the delay systems with additive time-varying delays system. The study on the system with two additive time-varying delay components has attracted many researchers since it was firstly proposed by [8]. Actually, the model has a strong application background in remote control and networked control (see, for instance [9–11], and the references therein). Generally, additive time-varying delays have different properties, it is no more applicable to study the stability condition for the system by using the approach to analyze delay systems with single delay [12–17]. Accordingly, it is of significance to consider stability for systems with two additive time-varying delay components.

The continuous dynamical systems with two additive time-varying delays was firstly introduced in [8], at the same time, the delay-dependent stability condition was provided. To improve the stability condition in [8,18] exploited more delayed

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state information by constructing the new Lyapunov functional, and more free weighting matrices were introduced to estimate the upper bound of the derivative of the Lyapunov functional. However, there leaves much room for improvement on the delay-dependent stability criteria in [8,18]. A new Lyapunov functional in [19] was constructed to derive new condition for the system. This paper take full advantage of any useful terms in the calculation of the time derivative. The delay information $d_1(t), d(t) - d_1(t), h - d(t)$ were not simply scaled as $h_1, h - h_1$ and h, respectively. Instead, the relationship that $d_1(t) + (h_1 - d_1(t)) = h_1,\ (h - d(t)) + (d(t) - d_1(t)) - (h_1 - d_1(t)) = h - h_1$ and $d(t) + (h - d(t)) = h$ were considered. And the stability conditions were in terms of many LMIs in [19]. However, many LMIs produced by a integral representation may brings conservative. A less conservative result given by [21] through constructing a new Lyapunov–Krasovskii functional and utilizing free matrix variables in approximating certain integral quadratic terms. However, the examples in [21] showed that its advantage results for given delay bound $\overline{\tau}_2$, but failing for given delay bound $\overline{\tau}_1$, compared to some other existing articles. The stabilization for the systems was considered by [24]. By using the convex polyhedron method, [22] handled the terms $d_1(t)N(Z_1+Z_2)^{-1}N^T,\ (h_1 - d_1(t))TZ_1^{-1}T^T,\ d_2(t)MZ_2^{-1}M^T$ and $(h-d(t))SZ_2^{-1}S^T$ to get stability conditions. The examples proved that the approach in [24] is less conservative than [8] and [18]. Combined with a reciprocally convex combination technique, the new stability condition was obtained [25], based on the novel constructed Lyapunov functional which fully considered the properties of the delay $d(t)$, $d_1(t)$ and $d_2(t)$. Recently, with novel Lyapunov functional, some improved stability conditions were provided for the systems in [26,27].

In fact, many stability conditions can be ultimately formed of some function of $d_1(t)$ and $d_2(t)$, instead of replacing those delays with their bounds in the process of proof, two very useful methods in [29,30] can effectively obtain the delay bounds results. However, to compute the condition functions about $d_1^2(t),\ d_2^2(t),\ d_1(t)d_2(t)$, $d_1(t)$ and $d_2(t)$ give the chance for us to provide the less conservative delay-dependent conditions. Generally, the results discussed above are mostly based on the Jensen's like inequalities which inevitably introduce some undesirable conservatism [33]. As a result, [33] presented a new class of inequalities, which produced tighter bounds than what the Jensen's like inequalities produced. Very recently, two less conservative inequalities were introduced based on the free-matrix ideal in [34,35]. They provided a better condition on the single integral inequality than [33]. On the other hand, recent research on double integral inequality and triple integral inequality in [2] proved that such these inequalities may play an important role in obtaining delay-dependent stability conditions for delay systems.

Motivated by the above discussion, we investigate, in this paper, the conventional delay systems with two additive time-varying delays to obtain delay-dependent stability condition. Firstly, many new inequalities are firstly introduced in six Lemmas. Secondly, with the idea of delay decomposition, a novel Lyapunov functional is constructed. Combining tighter estimation of the Lyapunov functional itself and its derivative with a reciprocally convex combination technique in [36,37], a new delay-dependent stability criterion is derived. Finally, two examples are given to show the effectiveness and the significant improvement of the proposed method.

Notation: Throughout this paper, a real symmetric matrix $P > 0 (\geq 0)$ denotes P being a positive definite (positive semi-definite) matrix. I is used to denote an identity matrix with proper dimension. Matrices, if not explicitly stated, are assumed to have compatible dimensions. The symmetric terms in a symmetric matrix are denoted by $\star$.

2. Preliminaries

Consider the delay systems with two additive time-varying delays as follows [8]

$$\dot{z}(t) = Az(t) + Bz(t - d_1(t) - d_2(t)) \tag{1}$$

where $z(t) \in R^n$ is the state vector. $A \in R^{n\times n}$, $B \in R^{n\times n}$ are constant matrices. The time delays $d_1(t)$ and $d_2(t)$ are time-varying differentiable functions that satisfy:

$$0 \leq d_1(t) \leq d_1, \quad 0 \leq d_2(t) \leq d_2 \tag{2}$$

and

$$0 \leq \dot{d}_1(t) \leq \mu_1 < \infty, \ 0 \leq \dot{d}_2(t) \leq \mu_2 < \infty \tag{3}$$

where d_1, d_2 and μ_1, μ_2 are constants. Naturally, we denote

$$d(t) = d_1(t) + d_2(t) \tag{4}$$

$$d = d_1 + d_2 \tag{5}$$

$$\mu = \mu_1 + \mu_2 \tag{6}$$

and μ is supposed as $\mu \leq 1$ in this paper.

In this paper, less conservative stability criteria for systems (1) satisfying conditions (2)–(3) will be proposed. Two novel introduced inequalities in the following technical Lemmas as well as some other inequalities will be used in the sequel.

Lemma 2.1 [2]. *For a given symmetric positive definite matrices $\mathcal{R} > 0$ and any differentiable function x in $[a, b] \to \mathbb{R}^n$, and $\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3 \in \mathbb{R}^{4n\times n}$, then the following inequality holds:*

$$-\int_a^b \int_\theta^b \dot{x}^T(s)\mathcal{R}\dot{x}(s)dsd\theta \leq \nu^T \Omega \nu, \tag{7}$$