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Magnetohydrodynamic stability of pressure-driven flow in an anisotropic porous channel: Accurate solution

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ABSTRACT

The stability of fully developed pressure-driven flow of an electrically conducting fluid through a channel filled with a saturated anisotropic porous medium is studied under the influence of a uniform transverse magnetic field using a modified Brinkman equation. An analogue of Squire's transformation is used to show that two-dimensional motions are more unstable than three-dimensional ones. The modified Orr–Sommerfeld equation for the problem is solved numerically and a more accurate solution is obtained using the Chebyshev collocation method combined with Newton's and golden section search methods. The critical Reynolds number R_c and the corresponding critical wave number α_c are computed for a wide range of porous parameter σ_p, the ratio of effective viscosity to the fluid viscosityΛ, the mechanical anisotropy parameter K_1, the porosity ε and the Hartman number M. It is found that the system remains unconditionally stable to small-amplitude disturbances for the Darcy case and the energy stability analysis is also performed to corroborate this fact.

1. Introduction

The stability of pressure-driven fluid flows in a channel is one of the classical problems in fluid mechanics and the copious literature available on this topic is well documented in the book by Drazin and Reid [1]. Its counterpart in a fluid-saturated porous channel has also been studied since transport processes through porous media play an important role in diverse applications, such as in petroleum industries, thermal insulation, chemical catalytic reactors, design of solid-matrix heat exchangers to mention a few [2–6].

The study of stability of Poiseuille flow in a horizontal channel under the influence of a magnetic field has also received considerable attention owing to its importance in a number of astrophysical contexts. Stuart [7] examined the stability of plane Poiseuille flow with a parallel magnetic field while Hains [8] investigated the influence of a coplanar magnetic field on the stability of a conducting fluid flowing between parallel planes. Potter and Kutchey [9] dealt with the problem discussed by Lock [10] and found that the fluid flow becomes more stable as the magnetic Prandtl number increases. Takashima [11] reconsidered the problem of Potter and Kutchey under the appropriate boundary conditions on the magnetic field perturbations. Makinde and Mhone [12] studied the temporal stability of magnetohydrodynamic (MHD) Jeffery–Hamel flows at very small magnetic Reynolds number while Proskurin and Sagalakov [13] analyzed the stability of Poiseuille flow in the presence of a longitudinal magnetic field.

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Nomenclature

(B_x, B_y, B_z)	components of magnetic induction $\vec{B}$
c	wave speed
c_r	phase velocity
c_i	growth rate
h	half-width of the porous channel
$(\hat{i}, \hat{j}, \hat{k})$	unit vectors in (x, y, z)-directions, respectively
$\underset{\sim}{K}$	second-order permeability tensor
K_1, K_2	mechanical anisotropy parameters
M	Hartmann number
P	total pressure
P_m	magnetic Prandtl number
R	Reynolds number
(u, v, w)	components of velocity vector $\vec{q}$
t	time
(x, y, z)	Cartesian co-ordinates

Greek symbols

α	horizontal wave number
β	vertical wave number
ε	porosity of the porous medium
Λ	ratio of effective viscosity to the fluid viscosity
μ	magnetic permeability
μ_e	effective viscosity
μ_f	fluid viscosity
ν	kinematic viscosity
σ	electrical conductivity
σ_p	porous parameter
ϕ	velocity stream function
ψ	magnetic stream function

Subscripts

b	basic state
c	critical

Moreover, MHD effects become important in the movement of conducting fluids through porous media. For example, MHD pumps are already in use in chemical energy technology for pumping electrically conducting fluids at some of the atomic energy centers and also in petroleum engineering. This type of problem also arises in electronic packages and microelectronic devices during their operation wherein it is possible to control the fluid flow due to electromagnetic forces [14–17]. Makinde and Mhone [18] examined numerically the temporal development of small disturbances in a channel filled with a saturated porous medium under the influence of magnetic field. Recently, Shankar et al. [19] investigated MHD stability of natural convection in a vertical porous slab.

In many technological applications of practical importance, hyper porous materials are being used which are usually formed by stretching many thin metal wires. Due to the structure of such solid material in which the fluid lies, there can be pronounced anisotropy in the permeability. Therefore, it is imperative to consider anisotropy in the permeability in discussing the stability of fluid flow through porous media. A detailed account on this topic can be found in the book by Nield and Bejan [20]. Moreover, Givler and Altobelli [21] experimentally demonstrated that for a high-permeability porous medium the Brinkman or effective viscosity is about 10 times the fluid viscosity. In view of this, it is proper to consider the ratio of these two viscosities as a separate parameter.

With these observations in mind, a general study has been undertaken here and investigated the stability of pressure-driven electrically conducting fluid flow in a channel filled with a high porosity anisotropic porous medium under the influence of a uniform transverse magnetic field. A more accurate solution to the stability eigenvalue problem is obtained. The outline of the paper is as follows: the mathematical formulation is given in Section 2, which contains the governing equations. Numerical solution adopted is given in Section 3. Results and discussion are presented in Section 4. Remarks and conclusions are reported in the last section.