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Numerical modeling of solidification process taking into account the effect of air gap

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A B S T R A C T

The paper presents a method of numerical modeling of binary alloy solidification process in a permanent mold in a two-dimensional area. The model takes into account the influence of the gas gap of variable width existing between the mold and the casting on the rate of solidification process. The numerical model is based on the finite element method (FEM) with independent spatial discretization of the casting and the mold. The displacements of these regions resulting from a thermally dependent changes of their volumes are also taken into account. The adopted approach involves the use of two separated meshes, where the temporary temperature fields are obtained. The thermal interaction between the casting and the mold is described by the appropriate boundary condition. The solution is obtained sequentially in each time step independently for the each region.

1. Introduction

Various technological processes such as welding, hardening or casting take into account thermal phenomena. Casting is a manufacturing process in which a molten material is poured into a solid mold and allowed to solidify. The casting and the mold are subjected to the large temperature changes. Cooled casting shrinks while the heat absorbing mold expands. The scale of this phenomenon depends on the thermophysical properties of the materials from which they are made. The casting and the mold remain in excellent thermal contact while the entire volume of the casting is in the liquid state. At this stage, the liquid material adapts to the changing shape of the mold. When the molten material starts to solidify it can contract away from the boundary of the mold due to shrinking. Heating up the mold causes its surface to expand. These two phenomena often support one another leading to the formation of the air gap between the casting and the mold. Mathematical descriptions of the non-ideal thermal contact can be found in the literature dating back more than 50 years [1]. Analytical solutions of the problem obtained with the use of the Fourier series was presented in [2]. The numerical solution of transient heat transfer between two semi-infinite flat regions based on the finite difference method (FDM) can be found in [3]. Nowadays the ideal and non-ideal thermal contact is the subject of interest of many scientists. It is required in modeling both the solidification problems where the mold and the casting are considered [4–6] as well in the problems of the heat flow in biological tissues [7–9]. The variation of the air gap width between the domains of different temperatures, especially during the numerical simulation of continuous casting process was discussed in [10–12].

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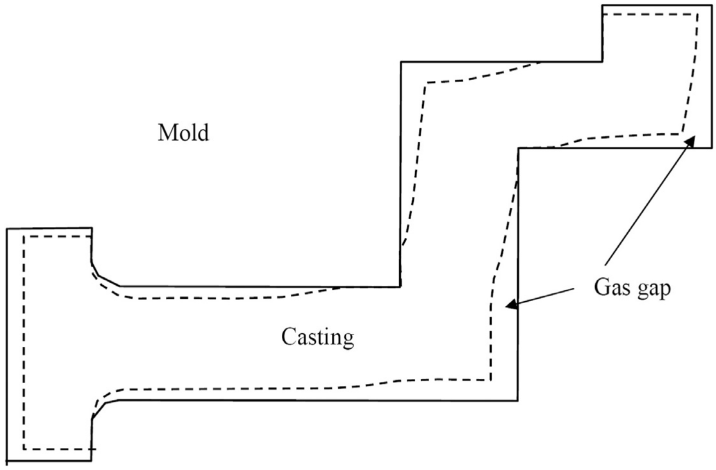

Fig. 1. Variable gap width in the casting of complex shape [13].

Assuming that the thermal processes in the casting and the mold are homogeneous one can estimate the size of forming gap d [m] depending on the diameter D [m] of the casting [13]:

$$d = [a_C(T_S - T_C) + a_M(T_M - T_{M0})]D \tag{1}$$

where a_C, a_M [K^{-1}] are the coefficients of thermal expansion of the casting and the mold, respectively, T_S [K] is the solidus temperature, T_C, T_M [K] represents the temperature of the casting and the mold interface, T_{M0} [K] is the initial temperature of the mold.

Eq. (1) indicates that the size of the gap is about 10 mm for a steel casting of 1 m diameter cooled to the room temperature, neglecting the expansion of the mold. It is significant gap causing serious thermal resistance. Unfortunately it is hard to predict the size of the gap with the use of Eq. (1) if the shape of the casting is complex or thermal expansion is non-homogeneous [14, 15]. In such case width of the gap is variable (Fig. 1). Eq. (1) also should not be used in the case of thin-walled castings where experimental measurements show no evidence for a gap [16]. Interesting approach using asymptotic methods for air gap investigation in the industrial continuous casting was presented in [17].

Another challenge is to determine the coefficient of thermal conductivity of the medium filling the gap. Colloquial term "air gap" is not in fact confirmed in the experimental studies [13]. Mold gases are rich in hydrogen which thermal conductivity is approximately 5.9 times greater than that of air. Assuming the gas in the gap is 50:50 mixture of hydrogen and air at 500 °C leads to the thermal conductivity 4 times higher than that of air.

The main goal of this study is to compare the three variants of numerical simulation of the solidification of bronze in the permanent mold made of cast iron. The first variant assumes the impact of the deformation of the casting and the mold on the formation of the gap. In the second case the gas gap is formed only as a result of shrinkage of the casting. The third variant of the calculation neglects thermal deformations which results in the ideal contact between considered regions.

2. Mathematical model

Fig. 2 schematically shows the considered problem. Entire region is divided into two parts. The first part, indicated by Ω_M is the mold made of cast iron. The mold is filled with solidifying bronze indicated by Ω_C. The heat of solidification is released during the liquid–solid transition of the alloy. In general, the mold and the casting are also subjected to thermal deformations. A gap appears between the internal fragments of boundaries Γ_M and Γ_C excluding the bottom where nearly perfect contact is observed. The local width of the gap h is variable according to the evolution of the mold and the casting temperatures. The direction of the most intense heat transfer is from the casting to the mold through the forming gap but the intensity is significantly affected by the increase of thermal resistance. Because the ambient temperature is lower than the mold and casting temperatures the heat is also transported outside the system through the external fragments of the boundaries Γ_M and Γ_C. The gravity forces and the mechanical loads are neglected in the model as well as the shrinkage during the liquid–solid phase change.

The governing equations of presented process are divided into two parts – one describing transport of the energy and solidification and second which describes thermal deformations of the mold and the casting.

Equation describing transient heat conduction in the mold:

$$\frac{\partial}{\partial x}\left(\lambda_M \frac{\partial T^M}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda_M \frac{\partial T^M}{\partial y}\right) = c_M \rho_M \frac{\partial T^M}{\partial t} \tag{2}$$