



Stationary distribution and extinction of a stochastic predator–prey model with additional food and nonlinear perturbation



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ABSTRACT

In this paper, we formulate and analyze a stochastic predator–prey model with additional food and nonlinear perturbation. Firstly, by constructing a suitable stochastic Lyapunov function, we establish sufficient conditions for the existence of an ergodic stationary distribution of the positive solution to the model. Then we obtain sufficient conditions for extinction of the predator species in two cases, one is the prey species surviving and the predator species extinction; the other is both the prey and predator species are extinct. Finally, some examples together with numerical simulations are provided to illustrate the analytical results.

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1. Introduction

In an ecosystem, most predator may not only eat a kind of food, for instance, wheat aphids and scale insect are foods of the coccinellid. Usually, we pay little attention to the effect of additional food (nonprey food) on the predator. The additional food provided to predator has influence on predator–prey systems [1–3], as they can decrease the amount of prey captured by predator. Recently, many biologist, experimentalists and theoreticians have considered the consequences of providing additional food to predators in predator–prey models [1–4]. For example, Huxel and McCann [1] studied the effect of additional food on the stability of a food web model. Srinivasu et al. [2] investigated the dynamical behavior of a predator–prey model in presence of additional food to the predator. Huxel et al. [3] considered a food web model with variable allochthonous inputs which are either one type available to both consumer and predator or two distinct types, one for consumer and the other for predator. Sahoo and Poria [4] reported a chaos control mechanism of a predator–prey system by incorporating additional food to predator.

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Habitat complexity is the structural complexity of habitats. Habitat complexity can mediate predator–prey interactions, affecting not only total predation rates, but also modifying selectivities for different prey species or size classes [5–8]. Pennings [9] and Grabowski [10] obtained that habitat complexity reduces encounter rates of predators with prey. Aquatic habitat has become structurally complex in presence of submerged vegetation or aquatic weeds [11]. In the laboratory systems of *Paramecium aurelia* (prey) and *Didinium nasutum* (predator), Luckinbill found that the time of coexistence of two species was prolonged by increasing the degree of habitat complexity using methyl cellulose in the Cerophyl medium (nutrient) [12]. Hence, it is interesting to incorporate the effect of habitat complexity when modeling predator–prey systems.

Recently, Sahoo and Poria [13] proposed and analyzed the following deterministic predator–prey model with additional food

$$\begin{cases} \frac{dx}{dt} = rx(1 - \frac{x}{k}) - \frac{a(1-c)xy}{1 + \alpha\xi + a(1-c)hx}, \\ \frac{dy}{dt} = \frac{\theta a(1-c)(x + \xi)y}{1 + \alpha\xi + a(1-c)hx} - dy, \end{cases} \tag{1.1}$$

where x denotes the density of prey, y is the density of the predator, r and k are intrinsic growth rate and environmental carrying capacity of the prey, respectively, a is the maximum rate of predation, c ($0 < c < 1$) measures the strength of habitat complexity, which reduces the predation rate, h is the half saturation value of the predator, α and ξ measure the quality and quantity of additional food, respectively, θ is the conversion efficiency of prey into the predator, d is the mortality rate of predator. The parameters are assumed to be positive constants. System (1.1) has three equilibria $E^0(0, 0)$, $E^1(k, 0)$ and the positive equilibrium $E^*(x^*, y^*)$, where

$$x^* = \frac{d(1 + \alpha\xi) - \theta a(1-c)\xi}{a(1-c)(\theta - hd)}, \quad y^* = \frac{r(k - x^*)[1 + \alpha\xi + a(1-c)hx^*]}{ak(1-c)}.$$

Considering the biological meaning, $E^*(x^*, y^*)$ needs to be satisfied the following conditions [13]:

$$0 < \xi < \frac{ka(1-c)(\theta - hd) - d}{\alpha d - a\theta(1-c)}, \quad c < 1 - \frac{d}{ka(\theta - hd)}, \quad hd + \frac{d}{ka} < \theta < 1.$$

The existence conditions of interior equilibrium point depends on quality and quantity of additional food supplied to predator.

On the other hand, it is well established that a real ecosystem is inevitably affected by the environmental noise [14,15]. May [16] revealed that due to continuous fluctuations in the environment, the birth rates, death rates, carrying capacities, competition coefficients and other parameters involved in the system should exhibit random fluctuation to a greater or lesser extent. Therefore the deterministic predator–prey model with additional food has some limitations to predict the future dynamics accurately and the stochastic one can make it [17–19]. Hence it is necessary to introduce random perturbation into the deterministic system. Here we introduce the nonlinear perturbation because the random perturbation may be dependent on square of the state variables x and y in system (1.1), respectively. Keeping this viewpoint in mind, the stochastic version corresponding to system (1.1) can be expressed as follows

$$\begin{cases} dx = [rx(1 - \frac{x}{k}) - \frac{a(1-c)xy}{1 + \alpha\xi + a(1-c)hx}]dt + x(\sigma_{11} + \sigma_{12}x)dB_1(t), \\ dy = [\frac{\theta a(1-c)(x + \xi)y}{1 + \alpha\xi + a(1-c)hx} - dy]dt + y(\sigma_{21} + \sigma_{22}y)dB_2(t), \end{cases} \tag{1.2}$$

where $B_i(t)$ are mutually independent standard Brownian motions, $\sigma_{ij}^2 > 0$ are the intensities of the white noise, $i, j = 1, 2$. Since stochastic system (1.2) is perturbed by nonlinear perturbation, the existence of ergodic stationary distribution is an interesting problem. However, as far as we know, there has been no result related this. In the present paper, we attempt to do some work in this field to fill the gap. Studying such problem is meaningful and challenging. The key difficulty is how to construct a suitable Lyapunov function and a bounded domain which is a subset of \mathbb{R}_+^2 .

This paper is organized as follows. In Section 2, we introduce some notations and lemmas concerning the existence of global positive solution and the ergodic stationary distribution. In Section 3, we establish sufficient conditions for the existence of a unique ergodic stationary distribution to system (1.2). In Section 4, sufficient conditions for extinction of the predator species in two cases are obtained, one is the prey species surviving and the predator species extinction; the other is both the prey and predator species are extinct. In Section 5, we introduce some examples and numerical simulations to confirm our main results. Finally, some concluding remarks and future directions are presented to close this paper.

2. Preliminaries

Throughout this paper, unless otherwise specified, let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t \geq 0}$ satisfying the usual conditions (i.e., it is increasing and right continuous while \mathcal{F}_0 contains all \mathbb{P} -null sets). Moreover, let $\mathbb{R}_+^n = \{(x_1, \dots, x_n) \in \mathbb{R}^n : x_i > 0, i = 1, \dots, n\}$. If f is a bounded function on $[0, +\infty)$, define $f^u = \sup_{t \in [0, +\infty)} f(t)$.

The following lemma concerns the existence and uniqueness of the global positive solution. Since the proof is standard (see [20]), we only present it without proof.

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