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Optimal superconvergence results for Volterra functional integral equations with proportional vanishing delays☆

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ABSTRACT

In this paper, we develop a new technique to study the optimal convergence orders of collocation methods for Volterra functional integral equations with vanishing delays on quasi-geometric meshes. Basing on a perturbation analysis, we show that for m collocation points, the global convergence order of the collocation solution is only m. However, the collocation solution may exhibit superconvergence with order $m+1$ at the collocation points. In particular, the local convergence order may attain $2m-1$ at the nodes, provided that the collocation is based on the m Radau II points. Finally, some numerical examples are performed to verify our theoretical results.

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1. Introduction

Consider the vanishing delay Volterra functional integral equation (VFIE)

$$u(t) = g(t) + b(t)u(qt) + \int_0^t K_1(t,s)u(s)ds + \int_0^{qt} K_2(t,s)u(s)ds, \quad t \in I := [0,T], \tag{1}$$

with $0<q<1$. Here, $g, b \in C(I)$, and the kernels $K_1(t, s)$ and $K_2(t, s)$ are continuous functions on $D := \{(t, s)\colon 0 \le s \le t \le T\}$ and $D_q := \{(t, s)\colon 0 \le s \le qt,\ t \in I\}$, respectively.

Many mathematical models in population dynamics, spread of epidemics and inverse problems in heat conduction can be described by the above pantograph equation [1–3]. The analysis of such equation dates back to the work of Volterra [4], where he showed that under certain conditions on the given data, the first kind integral equation

$$\int_{qt}^t H(t,s)u(s)ds = g(t), \quad t \in I, \quad 0<q<1, \tag{2}$$

can be converted into an equivalent second-kind VFIE of type (1).

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For the special case $b(t) \equiv 0$, Eq. (1) then reduces to a general Volterra integral equation (VIE)

$$u(t) = g(t) + \int_0^t K_1(t,s)u(s)ds + \int_0^{qt} K_2(t,s)u(s)ds, \qquad t \in I. \tag{3}$$

Since in our present work, the construction and the convergence analysis of collocation solution to (1) are closely related to the *non-vanishing* delay VFIE

$$\begin{cases} u(t) = f(t) + b(t)u(qt) + \int_{t_0}^t K_1(t,s)u(s)ds + \int_{t_0}^{qt} K_2(t,s)u(s)ds, & t \in (t_0, T], \\ u(t) = \psi(t), & t \in [0, t_0], \end{cases} \tag{4}$$

for convenience, we refer to (4) as the *non-vanishing* counterpart of (1) hereafter in this paper.

There are plenty of papers concerning the numerical results of the delay Volterra functional equations (see, i.e., [5–17] and references therein). Compared with the classical (non-delay) VIEs or Volterra integro-differential equations (VIDEs), the collocation solutions to the delay VIEs or VIDEs possibly suffer from a loss of convergence order on uniform meshes. For example, it is well known that if the collocation parameters are given by the m Gauss points, then the iterated collocation solution to (3) with $K_2(t,s) = 0$ possesses the local superconvergence order of $2m$ on uniform meshes. However, in 2005, Brunner et al. [18] proved that for the delay VIE (3), the local superconvergence order can only reach $m+2$. A similar phenomenon appears in the case of delay VIDE (compare [5, pp. 173–175] and [19]). As for delay VFIE (1), Xie et al. [20] showed that this convergence result is even worse. More precisely, the local superconvergence cannot occur on the underlying uniform meshes.

Fortunately, the above order reduction of the delay VIEs and VIDEs can be restored on certain non-uniform meshes. For example, under suitable chosen geometric meshes, Brunner et al. [19,21] showed that collocation with m Gauss points may lead to almost optimal order $2m - \varepsilon_N$ at the nodes, where $\varepsilon_N \to 0$ as the number of the nodes $N \to \infty$. Moreover, on quasi-geometric meshes, Brunner [5, pp. 328] pointed out that the 'classical' local superconvergence order $2m$ may exhibit again for the delay VIDEs, and the same is true for the delay VIE (3) (see [22]). Concerning the more general delay VFIE (1) with $b(t) \not\equiv 0$, whether the global or local convergence results can be improved on quasi-geometric meshes, however, is not yet understood, and this will be the focus of our present paper.

In this work, we first recast the *vanishing* delay equation (1) in the form of a corresponding *non-vanishing* delay problem. Unlike the usual method employed on quasi-geometric meshes, we now introduce a perturbation analysis method to investigate the attainable convergence orders of collocation solution to (1). As will be shown in Section 3, this new technique will greatly simplify the deducing of our theoretical convergence results. We remark that this work is different from our previous works [22,23]. In fact, in [23], we studied collocation methods for *non-vanishing* delay VFIEs. Also, the *vanishing* delay VIE (3) (i.e., Eq. (1) with $b(t) \equiv 0$) is considered in [22]. This work focuses on the *vanishing* delay VFIE (1) with $b(t) \not\equiv 0$, and the equation can be viewed as a generalization of that in [22].

The rest of this paper is organized as follows. In Section 2, an indirect collocation solution to (1) is constructed on quasi-geometric meshes. By a rigorous analysis, the optimal (global and local) convergence results of collocation solution to (1) are established in Section 3. Section 4 presents several numerical examples to validate the theoretical results given in the above section. Finally, some concluding remarks are provided in Section 5.

2. Construction of collocation solution

In this section, we will construct collocation solution to (1) on quasi-geometric meshes. Suppose that on certain small interval $[0, t_0] \subset I$, an approximation $\phi(t)$ to the analytic solution of (1) is already known and satisfies

$$||u - \phi||_{0,\infty} := \max_{t \in [0,t_0]} |u(t) - \phi(t)| \le C_0 t_0^{p_0} \tag{5}$$

for some positive integer p_0. Here, C_0 denotes a constant which is independent of t_0 or p_0.

For the above initial value $\phi(t)$, we then consider the following problem

$$\begin{cases} y(t) = \tilde{g}(t) + b(t)y(qt) + \int_{t_0}^t K_1(t,s)y(s)ds + \int_{t_0}^{qt} K_2(t,s)y(s)ds, & t \in (t_0, T], \\ y(t) = \phi(t), & t \in [0, t_0], \end{cases} \tag{6}$$

with

$$\tilde{g}(t) := g(t) + \int_0^{t_0} [K_1(t,s) + K_2(t,s)]\, \phi(s)ds.$$

It is readily to observe that if $\phi(t)$ coincides with the analytic solution $u(t)$ on $[0, t_0]$, Eq. (6) is then equivalent to (1) exactly.

Now, we are in a position to divide the computational interval $[0, T]$ on quasi-geometric meshes. Suppose that the initial interval $[0, t_0]$ is determined by setting

$$t_0 = q^M T \qquad \text{for} \quad \text{some} \quad M \in \mathbb{N},$$

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