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Existence of solitary solutions in systems of PDEs with multiplicative polynomial coupling

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ABSTRACT

Necessary conditions for the existence of solitary solutions to systems of nonlinear partial differential equations with multiplicative polynomial coupling are derived in this paper. The inverse balancing technique is used to explicitly determine necessary existence conditions in terms of orders of the system and the nonlinearity. As the orders of the system increase, the order of solitary solutions does not increase monotonically. A computational framework for the derivation of additional constraints on the parameters of higher-order solitary solutions is presented.

1. Introduction

Increased capabilities of symbolic computations have sparked a resurgence of interest in mathematical techniques for the construction of analytical solutions to partial differential equations (PDEs). In particular, a large amount of attention has been directed towards the construction of solitary (also called soliton) solutions due to their significance in research fields ranging from physics and engineering to biology and medicine [1–3].

Solitary solutions are one of the primary tools used in the investigation of various real-world problems. Soliton behaviour in a reaction-diffusion system of PDEs applied for the simulation of myocardial beats is studied in [4]. Dynamics of solitary waves in water channels with sharp bends and branching points is considered in [5]. Models of superfluidity and superconductivity based on solitary waves are presented in [6,7]. It has recently been shown in [8] that tumour-induced angiogenesis can be controlled through solitons that are driving the process. Travelling solitary waves in nonlinear viscoelastic solids are discussed in [9].

Coupled systems of PDEs are considered in this paper. Typical examples of the analysis such systems in various fields of research are given below. A technique for despeckling ultrasound images based on coupled selective degenerate diffusion PDEs is presented in [10]. Steel hardering process is mathematically modeled using coupled PDEs describing temperature and carbon concentration in [11]. A parabolic-hyperbolic coupled system of Stokes-Lame PDEs governing fluid-structure interaction is considered in [12]. The Dirichlet problem for a class of non-linear coupled systems of reaction-diffusion PDEs is studied in [13].

A number of novel mathematical methods have been used to analyze solitary solutions to coupled PDEs in recent years. Soliton and soliton-like solutions of the coupled Schrödinger equations are studied using numerical and variational

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techniques in [14]. Orbital stability of solitary wave solutions to three-coupled long wave-short wave interaction equations is investigated in [15]. Multi-soliton solutions to the coupled Korteweg-de-Vries (KdV) equations are constructed in [16]. The Darboux transformation is used to construct soliton, breather and rogue wave solutions to the coupled Fokas-Lenells PDE system in [17]. The asymptotic stability of solitary wave solutions to the Schrödinger equation coupled with nonlinear oscillators is investigated in [18].

The main objective of this paper is to derive necessary existence conditions for solitary solutions to the following system of PDEs coupled with multiplicative polynomial terms:

$$\sum_{j=1}^{n}\sum_{s=0}^{j} a_{s,j-s}\frac{\partial^j \mathcal{U}}{\partial x^s \partial t^{j-s}} = \sum_{j=0}^{k}\sum_{s=0}^{j} b_{j-s,s}\mathcal{U}^{j-s}\mathcal{V}^s; \tag{1}$$

$$\sum_{j=1}^{m}\sum_{s=0}^{j} c_{s,j-s}\frac{\partial^j \mathcal{V}}{\partial x^s \partial t^{j-s}} = \sum_{j=0}^{l}\sum_{s=0}^{j} d_{j-s,s}\mathcal{U}^{j-s}\mathcal{V}^s, \tag{2}$$

where $\mathcal{U} = \mathcal{U}(t,x)$, $\mathcal{V} = \mathcal{V}(t,x)$; $n, m, k, l \in \mathbb{N}$; coefficients $a_{s,j-s}, b_{j-s,s}, c_{s,j-s}, d_{j-s,s} \in \mathbb{R}$.

The standard independent variable substitution $z := \alpha x + \beta t; \alpha, \beta \neq 0$ transforms (1), (2) into a system of ordinary differential equations (ODEs) with multiplicative polynomial coupling:

$$u_z^{(n)} + a_{n-1}u_z^{(n-1)} + \cdots + a_1 u_z' = \sum_{j=0}^{k} Q_j^{(b)}(u,v); \tag{3}$$

$$v_z^{(m)} + c_{m-1}v_z^{(m-1)} + \cdots + c_1 v_z' = \sum_{j=0}^{l} Q_j^{(d)}(u,v), \tag{4}$$

where $u = u(z) = \mathcal{U}(t,x)$; $v = v(z) = \mathcal{V}(t,x)$; coefficients a_j, $j = 1,\dots,n-1$; c_j, $j = 1,\dots,m-1$ are linear combinations of $a_{s,j-s}$ and $c_{s,j-s}$ respectively. Functions $Q_j^{(b)}, Q_j^{(d)}$ read:

$$Q_j^{(b)}(u,v) := \sum_{s=0}^{j} b_{j-s,s}u^{j-s}v^s; \tag{5}$$

$$Q_j^{(d)}(u,v) := \sum_{s=0}^{j} d_{j-s,s}u^{j-s}v^s. \tag{6}$$

Solitary solutions of the following form are considered:

$$u_0 = \sigma\frac{\prod_{j=1}^{N}\left(\exp\left(\eta(z-z_0)\right) - u_j\right)}{\prod_{j=1}^{N}\left(\exp\left(\eta(z-z_0)\right) - z_j\right)}; \tag{7}$$

$$v_0 = \gamma\frac{\prod_{j=1}^{N}\left(\exp\left(\eta(z-z_0)\right) - v_j\right)}{\prod_{j=1}^{N}\left(\exp\left(\eta(z-z_0)\right) - z_j\right)}, \tag{8}$$

where $N \in \mathbb{N}$; $\sigma, \gamma, \eta \in \mathbb{R}$; $u_j, v_j, z_j \in \mathbb{C}$; $u_s \neq z_j$; $v_s \neq z_j$; $s, j = 1,\dots,N$.

The inverse balancing technique is used to determine necessary existence conditions of solitary solutions in terms of the system's derivative orders (n, m), nonlinearity orders (k, l) and solitary solution order N. The underlying idea of this technique is the assumption that the solution parameters are known and the system parameters can be expressed in terms of the solution parameters. Since systems (3), (4) is linear in coefficients a_j, $b_{j,s}$, c_j, $d_{j,s}$, the inverse balancing procedure results in a system of linear algebraic equations with respect to a_j, $b_{j,s}$, c_j, $d_{j,s}$. The necessary conditions for the existence of (7), (8) in (3), (4) are equivalent to non-degeneracy conditions of the obtained algebraic linear system.