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Application of wavelet collocation method for hyperbolic partial differential equations via matrices

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ABSTRACT

In this work, we developed an efficient computational method based on Legendre and Chebyshev wavelets to find an approximate solution of one dimensional hyperbolic partial differential equations (HPDEs) with the given initial conditions. The operational matrices of integration for Legendre and Chebyshev wavelets are derived and utilized to transform the given PDE into the linear system of equations by combining collocation method. Convergence analysis and error estimation associated to the presented idea are also investigated under several mild conditions. Numerical experiments confirm that the proposed method has good accuracy and efficiency. Moreover, the use of Legendre and Chebyshev wavelets are found to be accurate, simple and fast.

1. Introduction

Wavelet theory, developed mostly over the last 30 years, has generated a tremendous interest in many areas of research in sciences and engineering [1–6]. However, most of the applications of wavelets have been focused on analyzing the data and using the wavelets as a tool for data compression [7,8]. In recent years, solving partial differential equations (PDEs) with wavelets has received considerable attention among many researchers. Most of the physical problems like heat conduction, wave propagation, laser beam models are modeled as PDEs whose solutions cannot be easily obtained by the classical methods. This may be due to either the nonlinearity associated with the equation or inappropriate solution space since PDEs are usually applied for simulating the physical phenomena in many branches of sciences and engineering. Therefore the solution of PDEs should be considered in the best manner for extracting the behavior of unknown variables that are formulated under the considerable models. On the other hand, only for some special classes of PDEs analytical solutions are available and in many cases it is impossible to obtain the analytical solutions. Therefore, a wide class of numerical methods were introduced such as spectral method [9], finite difference methods (FDMs), finite element methods (FEMs) (for instance see [10] and the references therein). If the solution of physical problem has regular features, any of these numerical techniques can be applied. However, in many physical problems there exists a multiplicity of very different spatial and temporal scales in the solution, as in strongly time-dependent non-Newtonian convection, formation of shock waves in compressible gas flow, pattern formation in hydrodynamics systems and turbulent flow around bluff bodies. This particular attribute of multiple spatial scales, which possibly change over time will put great strain on these numerical methods. Spectral methods have problems capturing large irregularities of the solution. The main difficulties of existing adaptive finite difference methods or finite element methods is developing computationally efficient robust adaptive procedure, which dynamically adapts the

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computational grid to local structure of the solution. Therefore, the application of the methods based on wavelets for the numerical solution of PDEs has recently been studied from the theoretical and the computational point of view in many papers in which several good features of such methods have been described [11–14]. A crucial role in design of such methods is played by the good localization properties that wavelet displays both in the space and frequency. Good localization properties in physical and wavenumber spaces are to be contrasted with the spectral approach, which employs infinitely differentiable functions but with global support and small discrete changes in the resolution. On the other hand, finite-difference, finite-volume and finite-element methods use basis with small compact support but poor continuity properties. In this research article, we proposed an efficient wavelet collocation method (WCM) based on Legendre and Chebyshev wavelets for the approximate solution of first order HPDEs of the following form [15]

$$\frac{\partial u(x,t)}{\partial t}+\frac{\partial u(x,t)}{\partial x}+au(x,t)=bv(x,t)\quad (x,t)\in\Lambda(=[0,1]\times[0,1]) \tag{1}$$

with the initial conditions $u(0,t)=f(t)$ and $u(x,0)=g(x)$.

Also, we assume that a and b are the real scalars, f, g, v are the known functions and u is the unknown function which should be determined.

HPDEs are one of the most important subclasses of PDEs. Many of the mechanic equations are hyperbolic, and so the study of hyperbolic equations is of substantial contemporary interest. HPDEs can model the vibration of structures (e.g. buildings, machines and beams) and are the basis for fundamental equations of atomic physics. Wave equations and Telegraph equations are the typical type examples of HPDEs. Telegraph equation [16] is widely used in signal analysis for transmission and propagation of electric signals and also has many other applications.

Tohidi and Toutounian [15] solved Eq. (1) for the matrix form by Bernoulli matrix approach. Also the numerical study of such equations in the case of scalar parameters was considered by many researchers such as [17–19]. Moreover, approximate investigation of some special classes of Eq. (1) was discussed in the literature such as [20].

We have introduced an efficient WCM based on the operational matrices of integration of Legendre and Chebyshev wavelets for the approximate solution of Eq. (1). The advantages of using Legendre and Chebyshev wavelets proposed in this work are the properties of their basis functions. Orthogonal polynomials are used to construct the basis functions. Moreover, the Legendre and Chebyshev wavelets basis can combine the advantages of both infinitely differentiable functions and small compact support. In contrast to other methods, wavelet methods along with their operational matrices are easy to extend to multidimensional problems. Operational matrices of integration of Legendre and Chebyshev wavelets combining with collocation method are used to reduce the solution of Eq. (1) into the linear algebraic system of equations. Application of operational matrices of integration in which the numerical solution of PDEs was obtained, can be found in [20,21] in the case of Walsh and Berstein approximation, respectively.

The remainder of the paper is organized as follows; we give some brief review of Legendre and Chebyshev wavelets, their approximation properties and function approximation in Section 2. Operational matrices of integration of Legendre and Chebyshev wavelets are introduced in Section 3. WCM based on Legendre and Chebyshev wavelets for solving Eq. (1) is established in Section 4. In Section 5, convergence analysis and error estimation for the proposed method are given under the several mild conditions. Numerical experiment is discussed in Section 6. Finally, some concluding remarks are given in Section 7.

2. Definitions

2.1. One dimensional Legendre wavelets

Wavelets constitute a family of functions constructed from dilation and translation of single function called the mother wavelet. When the dilation parameter a and the translation parameter b vary continuously, we have the following family of continuous wavelets [22]:

$$\psi^L_{a,b}(t)=|a|^{-\frac{1}{2}}\psi^L\left(\frac{t-b}{a}\right),\, a,b\in\mathbf{R},\, a\neq 0.$$

If the parameters a and b are restricted to the discrete values as $a=a_0^{-k}, b=nb_0a_0^{-k}, a_0>1, b_0>0,$ n and k are positive integers, we have the following family of discrete wavelets:

$$\psi_{k,n}(t)=|a_0|^{-\frac{k}{2}}\psi\left(a_0^kt-nb_0\right),$$

where $\psi_{k,\,n}(t)$ form a wavelet basis for $L^2(\mathbf{R})$. In particular, when $a_0=2$ and $b_0=1$ then $\psi_{k,\,n}(t)$ form an orthonormal basis.

Legendre wavelets $\psi^L_{n,m}(t)=\psi(k,\widehat{n},m,t)$ have four arguments; $\widehat{n}=2n-1, n=1,2,3,\ldots,2^{k-1}$, k can assume any positive integer, m is the order of Legendre polynomials and t is the normalized time. One dimension Legendre wavelets over [0,1] defined as follows:

$$\psi^L_{n,m}(t)=\begin{cases}\sqrt{m+\frac{1}{2}}\,2^{\frac{k}{2}}p_m(2^kx-2n+1), & \frac{(n-1)}{2^{k-1}}\leq x\leq\frac{n}{2^{k-1}},\\ 0, \quad \text{Otherwise.}\end{cases} \tag{2}$$