Applied Mathematics and Computation 320 (2018) 524–534

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Asymptotic mean-square boundedness of the numerical solutions of stochastic age-dependent population equations with Poisson jumps

Yongzhen Pei[a,*], Hongfu Yang[b], Qimin Zhang[c], Fangfang Shen[a]

[a] School of Computer Science and Software Engineering, Tianjin Polytechnic University, Tianjin 300387, China
[b] School of Mathematics and Statistics, Northeast Normal University, Changchun, Jilin 130024, China
[c] School of Mathematics and Computer Science, Ningxia Univerity, Yinchuan 750021, China

ARTICLE INFO

Keywords:
Asymptotic mean-square boundedness
Compensated numerical methods
Non-compensated numerical methods
Stochastic age-dependent population equations
Poisson jumps

ABSTRACT

This paper focuses on asymptotic mean-square boundedness of several numerical methods applied to a class of stochastic age-dependent population equations with Poisson jumps. The conditions under which the underlying systems are asymptotic mean-square boundedness are considered. It is shown that the asymptotic mean-square boundedness is preserved by the compensated split-step backward Euler method and compensated backward Euler method without any restriction on stepsize, while the split-step backward Euler method and backward Euler method could reproduce asymptotic mean-square boundedness under a stepsize constraint. The results indicate that compensated numerical methods achieve superiority over non-compensated numerical methods in terms of asymptotic mean-square boundedness. Finally, an example is given for illustration.

1. Introduction

Stochastic differential equations can be found in many applications in such areas as economics, biology, finance, ecology, and other sciences [1–6]. Asymptotic properties of the solutions of stochastic population equations have been widely studied in the past decades, particularly the stability theory has been attracted lots of attention. For example, Ma et al. [7] studied the asymptotic stability of stochastic age-dependent population equations with Markovian switching. Zhang [8] showed the exponential stability of numerical solutions to stochastic age-structured population system with diffusion. Ma et al. [9] analyzed the asymptotic behavior of energy solutions to stochastic age-dependent population equations driven by Lévy processes.

Another important asymptotic property of the stochastic age-dependent population equations solutions, the asymptotic boundedness has its own right. Unlike the stability property that requires the solutions be attracted by an equilibrium state, the boundedness property only requires the solutions stay within certain regime as time tends to infinity [10].

On the other hand, Poisson jumps are becoming popular to models which have many burst phenomenon. In the stochastic age-dependent population system, due to some brusque variation from some sudden events, for example tsunami, earthquakes, impacts of extraterrestrial objects and so on, the size of the population systems increases or decreases

* Corresponding author.
E-mail addresses: yongzhenpei@163.com, peiyongzhen@sina.com (Y. Pei), yanghf783@nenu.edu.cn, hongfu_yang@sina.com (H. Yang), zhangqimin64@sina.com (Q. Zhang), shenfangfang1989@163.com (F. Shen).

https://doi.org/10.1016/j.amc.2017.10.030
0096-3003/

drastically, so it is better to describe the dynamics of population density by using the jump-diffusion system. Consider a class of stochastic age-dependent population system with Poisson jumps of the form

$$\begin{cases} d_tP_t = -\frac{\partial P_t}{\partial a}dt - \mu(t,a)P_tdt + f(t,P_t)dt + g(t,P_t)dB(t) + h(t,P_t)dN(t), & \text{in } I, \\ P(0,a) = P_0(a), & \text{in } [0,A], \\ P(t,0) = \int_0^A \beta(t,a)P(t,a)da, & \text{in } [0,T], \end{cases} \tag{1.1}$$

where $I=(0,A)\times(0,T), P_0 := P(0,a)$, d_tP_t is the differential of P_t with respect to t, i.e., $d_tP_t = \frac{\partial P_t}{\partial t}dt$. $P_t := P(t, a)$ denotes the population density of age a at time t, $\beta(t, a)$ denotes the fertility rate of females of age a at time t, $\mu(t, a)$ denotes the mortality rate of age a at time t. $f(t, P_t)$ effects of external environment for population system, $g(t, P_t)$ is a diffusion coefficient, $h(t, P_t)$ is a jump coefficient. Let $N(t)$ be a scalar Poisson process with intensity $\lambda > 0$ which is independent of Brownian motion $B(t)$.

However, due to the difficulty to find the explicit solutions for jump diffusion system (1.1), it is necessary to develop numerical methods and study the properties of numerical solutions for system (1.1). For example, Li et al. [11] studied the Euler numerical method for stochastic age-dependent population equations with Poisson jumps. Wang et al. [12] analyzed the convergence of the semi-implicit Euler method for stochastic age-dependent population equations with Poisson jumps. Rathinasamy et al. [13] developed the numerical method for stochastic age-dependent population equations with Poisson jump and phase semi-Markovian switching. Tan et al. [14] investigated the convergence of the split-step θ-method for stochastic age-dependent population equations with Poisson jumps.

Although [7–9,11–16] have studied the convergence and stability for stochastic age-dependent population systems in recent years, but there are few papers investigating the asymptotic boundedness of the numerical methods. In this paper, we shall extend the idea from the papers [1,4,14] to the asymptotic mean-square boundedness of the numerical solutions for jump diffusion system (1.1). This work differs from existing results (see e.g. Li et al. [11], Wang et al. [12], Rathinasamy et al. [13] and Tan et al. [14]) in that (i) compensated and non-compensated numerical methods are considered and (ii) asymptotic mean-square boundedness is involved.

The contents of this paper are as follows: In Section 2 we give preliminaries. In Section 3, some criterion for asymptotic mean-square boundedness of (1.1) are shown. Section 4 discusses asymptotic mean-square boundedness of split-step backward Euler method and compensated split-step backward Euler method. Section 5 is devoted to prove that backward Euler method and compensated backward Euler method can inherit the asymptotic mean-square boundedness of (1.1). Finally, we give an example to illustrate the main results in Section 6.

2. Preliminaries

Let

$$V = H^1([0,A]) \equiv \Big\{\varphi | \varphi \in L^2([0,A]), \frac{\partial \varphi}{\partial a} \in L^2([0,A]),$$
$$\text{where } \frac{\partial \varphi}{\partial a} \text{ is generalized partial derivatives}\Big\}.$$

V is a Sobolev space. $H = L^2([0,A])$ such that

$$V \hookrightarrow H \equiv H' \hookrightarrow V'.$$

V' is the dual space of V. We denote by $\|\cdot\|$, $|\cdot|$ and $\|\cdot\|_*$ the norms in V, H and V' respectively; by $(\cdot,\cdot)$ the scalar product in H, and by $\langle\cdot,\cdot\rangle$ the duality product between V and V', i.e.,

$$\langle u, v\rangle = \int_0^A u\cdot v da, \quad u\in V, \quad v \in V'.$$

For an operator $G \in \mathcal{L}(K, H)$ being the space of all bounded linear operators from K into H, we denote by $\|G\|_2$ its Hilbert–Schmidt norm, i.e.

$$\|G\|_2^2 = \text{tr}(GQG^T),$$

where Q is increment covariance operator of $B(t)$. Let $C = C([0,T];H)$ be the space of all continuous function from [0, T] into H with sup-norm $\|\psi\|_C = \sup_{0\le s\le T} |\psi(s)|$, $L_V^p = L^p([0,T];V)$ and $L_H^p = L^p([0,T];H)$.

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\ge 0}$, satisfying the usual conditions (i.e., it is increasing and right continuous while $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets). Let $f(t,\cdot)$, $h(t,\cdot) : L_H^2 \to H$ be a family of nonlinear operators, $\mathcal{F}_t$-measurable almost surely in t. $g(t,\cdot) : L_H^2 \to \mathcal{L}(K, H)$ be a family of nonlinear operator, $\mathcal{F}_t$-measurable almost surely in t. $P_0 \in L_H^2$.

In order to analyse the boundedness of jump diffusion system (1.1), we impose the following standard hypotheses:

(A2.1) $\mu(t, a)$, $\beta(t, a)$ are nonnegative measurable, and

$$\begin{cases} 0 \le \bar{\mu} \le \mu(t,a) < \infty & \text{in } I, \\ 0 \le \beta(t,a) \le \bar{\beta} < \infty & \text{in } I. \end{cases}$$