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Controller design for time-delay system with stochastic disturbance and actuator saturation via a new criterion

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ABSTRACT

This paper deals with the problem of controller design for time-delay system with stochastic disturbance and actuator saturation. By use of more appropriate Lyapunov–Krasovskii functional (LKF) and a new criterion for the domain of attraction, less conservative conditions for stochastic stability are proposed. Then, the difficulties of the domain of attraction confronted in system analysis and synthesis can be overcome. These sufficient conditions are derived in terms of linear matrix inequality (LMI). Finally, two practical examples demonstrate the validity of the given results.

1. Introduction

Time delay exists widely in practical systems, such as electrical signals transmission in long lines, temperature control system, jet aircraft, ship orientation instrument, and fluid flow in the pipeline [1–7]. With the progress of modern technology, it is inevitably that the existence of time delay is an important factor to make performance worse and system out of control, which increases the complexity of theoretical analysis and engineering application, and also brings challenges to controller design. The study of time delay is categorized into two systems: delay-dependent and delay-independent system. Delay-dependent system means that the information on the size of delay is known while the delay-independent system is assumed to be time-delay unknown or possibly unbounded. Since the delay-dependent system is less conservative than the delay-independent one, more effort has been devoted to delay-dependent systems; for details, see [8–25]. At the same time, it is inevitable that stochastic phenomenon widely exists in practical engineering and a great number of control issues concerning stochastic systems have been studied in many branches of science and industry such as chemical engineering, biological system, flight control systems [26], air traffic management [27], communication networks [28], biology systems [29], economic system [30], population growth model [31], and other systems [32–40].

On the other hand, actuator saturation is often inevitable in many practical applications, which may deteriorate the performance of dynamic system and even causes the instability of closed-loop system. Recently, it is noted that saturated system has gained particular research interests, such as *RLC* series circuit [41], balancing pointer [42,43], cart-spring-pendulum system [42,44], F-8 aircraft system [42,45], and control theory [42]. In fact, linear system with actuator saturation owes the properties of the linear and nonlinear systems. Even if the open-loop system is linear, the presence of the actuator saturation

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makes the closed-loop system nonlinear. In such case, we should be aware of the nonlinear effects produced by actuator saturation on dynamic performance. Over the last decades, a great deal of attention has been devoted to analysis and synthesis of dynamic system with actuator saturation in [46–51]. In general, there are two methods to deal with saturation. One is to deal with the saturation directly by using the polytopic model for representing the saturated closed-loop system which can be described by linear system with control constraint; the other (usually called anti-windup design) is to deal with the saturation indirectly, whose thought is to construct an output feedback controller that satisfies the performance specifications without considering actuator saturation, and then design an anti-windup compensator to weaken the adverse effect of actuator saturation.

This paper will study controller design for time-delay system with actuator saturation and stochastic disturbance via a new criterion. It is necessary to point out the differences between the present work and existing relative works [8,9,35,48,49]. First, the stochastic disturbance was not considered in [8,9,48,49] while the model in this paper includes the stochastic disturbance. Second, the derivative of time delay in [35] is given as $\dot{h}(t) \le \mu < 1$ while the derivative of time delay in this paper is given as $\dot{h}(t) \le \mu$. For time-delay system with stochastic disturbance and actuator saturation, the problem of removing restrictive conditions on time delay, choosing the more appropriate LKF, and expanding the domain of attraction will be more complicated and challenging. Moreover, the problem of controller design for time-delay system with stochastic disturbance and actuator saturation has not been fully investigated, which motivates the present study. The contributions of this paper are listed as follows: (i) By defining a new criterion for the domain of attraction and constructing more appropriate LKF, less conservative conditions for the underlying system are proposed; (ii) A controller is designed and the domain of attraction of system is expanded; (iii) Two practical examples about the *RLC* series circuit model and the single-link robot arm model illustrate the validity of the obtained results.

The remainder of this paper is organized as follows. In Section 2, the system formulation and some necessary lemmas are given. Section 3 and Section 4 are devoted to deriving the results on stochastic stability analysis, controller design, and estimation of the domain of attraction. Some examples are provided to illustrate the feasibility of the obtained results in Section 5. Concluding remarks are given in Section 6.

Notations

- $\mathcal{N}^T$: transpose of $\mathcal{N}$;
- $\mathbb{R}^n$: n dimensional Euclidean space;
- $\mathbb{R}^{n\times m}$: set of $n \times m$ real matrices;
- $\mathcal{E}\{\cdot\}$: mathematical expectation;
- $*$: an ellipsis for symmetry;
- $\Lambda[-h, 0]$: space of the continuously differentiable vector functions $\varphi(\theta)$ over $[-h, 0]$;
- $\|\varphi\|_c = sup_{-\tau\le\theta\le 0}\|\varphi(\theta)\|$: norm of $\varphi \in \Lambda[-h, 0]$.

2. Problem statement and preliminaries

Consider the following time-delay system with actuator saturation and stochastic disturbance

$$\begin{aligned} dx(t) =& [\mathcal{A}x(t) + \mathcal{A}_d x(t-h(t)) + \mathcal{B}\sigma(u(t))]dt \\ &+ [\mathcal{W}x(t) + \mathcal{W}_d x(t-h(t))]dw(t), \\ x(t_0+\theta) =& \varphi(\theta), \forall\theta \in [-h, 0], \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and $w(t) \in \mathbb{R}$ are the state vector and the standard Wiener process; $\sigma(\cdot) : \mathbb{R}^m \to \mathbb{R}^m$ is the vector-valued standard saturation function defined as $\sigma(u) = [\sigma^T(u_1),\ \sigma^T(u_2), \ldots,\ \sigma^T(u_m)]^T$, where $\sigma(u_q) = sign(u_q)min\{\bar{u}_q, |u_q|\},\ q = 1, 2, \ldots, m$ with sign function. $\mathcal{A},\ \mathcal{A}_d,\ \mathcal{B},\ \mathcal{W}$, and $\mathcal{W}_d$ are known constant matrices with appropriate dimensions.

The controller is given as

$$u(t) = \mathcal{K}x(t), \tag{2}$$

where K is the gain matrix to be designed.

The time-varying delay $h(t)$ satisfies

$$0 \le h(t) \le h, \dot{h}(t) \le \mu, \tag{3}$$

where h and μ are known real constant scalars.

Remark 1. For time-delay system with actuator saturation and stochastic disturbance [35], the time delay is given as $\dot{h}(t) \le \mu < 1$, which may have some conservativeness. Here, the time-varying delay (3) is more general.

Consider the symmetric polyhedron as $\Psi(\mathcal{F}) = \{x(t) \in \mathbb{R}^n : |\mathcal{F}_q x(t)| \le \bar{u}_q, q = 1, 2, \ldots, m\}$, where $\mathcal{F}_q$ stands for the qth row of the matrix $\mathcal{F}$. Consider an ellipsoid given as $\eta(\mathcal{P}) = \{x(t) \in \mathbb{R}^n : x^T(t)\mathcal{P}x(t) \le 1\}$, where $\mathcal{P} > 0$. $\wp$ stands for the set of $m \times m$ diagonal matrices, in which the elements are either 1 or 0. Define every element of $\wp$ as $\mathcal{U}_v,\ v = 1, 2, \ldots, 2^m$ and $\mathcal{U}_v^{-1} = I - \mathcal{U}_v$. Obviously, if $\mathcal{U}_v \in \wp$, then $\mathcal{U}_v^- \in \wp$