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Vibrational responses of antisymmetric angle-ply laminated conical shell by the methods of polynomial based differential quadrature and Fourier expansion based differential quadrature

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ABSTRACT

Vibrational response of antisymmetric angle-ply laminated conical shell based on transverse shear deformation theory is analyzed in this research. The equilibrium and corresponding boundary equations including the extension-coupling terms are derived in terms of displacement and rotational components. Two types of differential quadrature methods (DQM) including polynomial based differential quadrature (PDQ) and Fourier expansion based differential quadrature (FDQ) are applied to convert the existing differential equations to algebraic equations. After discretization of existing differential equations, the eigenvalue equation system for the functional value in the whole domain is established and solved. A series of numerical tests are carried out to validate the present approach that highly accurate results are obtained only few grid points. Finally, the parametric studies are performed to illustrate the effects of different boundary conditions, fiber angle and number of layers and also various aspect ratio of the shell including thickness to radius ratio length to radius on the frequency parameter.

1. Introduction

The conical shell structures play significant contribution in diverse industrial applications as structural components, such as nuclear, aeronautical, automobile, and submarine and other industrial fields. On the other hand, the angles-ply composite laminated shells with proper adjustment can boost specific strength and stiffness and avoid premature crack and corrosion in structures. In this regard, analyzing these kinds of shells with aforementioned characterizations is the most important step in optimum mechanical engineering designing. Vibration analysis of composite laminated composite shells are carried out using different numerical approaches. As compared to the other numerical technique such as finite difference, finite element methods, the two version of DQ method including (PDQ) and (FDQ), can achieve very precise numerical results using considerably smaller grid points and therefore, needing little computational efforts. DQ numerical procedure based on PDQ and FDQ is well developed and employed to diverse application fields of engineering and physical sciences and can confront any problem with complex geometrical and boundary conditions. In this section a series of papers corresponding to vibration

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Nomenclature

$\kappa_x,\ \kappa_\theta,\ \kappa_{x\theta}$	curvatures
$u,\ v,\ w$	displacement components in meridian, circumferential and normal
$u_0,\ v_0,\ w_0$	displacement components in meridian, circumferential and normal at mid-surface
$E_{11},\ E_{22},\ \nu_{12},\ \nu_{23},\ G_{12},\ G_{13},\ G_{23}$	engineering constants of single lamina where subscripts 1 and 2 represent directions parallel and transverse to fiber
$A_{ij},\ B_{ij},\ D_{ij}$	extensional, coupling and bending stiffness
L	length of cone along its generator
ρ	mass density
ω	natural frequency
λ	non-dimensional frequency parameter
$R1$	radius of the cone at small edge
$R2$	radius of the cone at large edge
γ	semi-vertex angle of cone
$\psi_x,\ \psi_\theta$	shear rotation of any point on the mid surface normal to xz and θ–z planes
$(x,\ \theta,\ z)$	shell coordinate
$\varepsilon_{xx},\ \varepsilon_{\theta\theta},\ \varepsilon_{\theta z},\ \varepsilon_{xz},\ \varepsilon_{x\theta}$	strain components
$\sigma_x,\ \sigma_\theta,\ \tau_{x\theta},\ \tau_{xz},\ \tau_{\theta z}$	stress components
$\bar{Q}_{ij}$	stiffness coefficients of a single lamina related to the x, θ, z
Q_{ij}	stiffness coefficients of a single lamina related to material principle axis
$N_\theta,\ N_x,\ N_{x\theta}$	the total In-plane force resultants
$M_x,\ M_\theta,\ M_{x\theta}$	the total bending moment resultants
$Q_x,\ Q_\theta$	transverse shear force in x and θ direction
$\eta^n,\ C^n$	the weighting coefficients for the nth-order derivatives in x and θ directions
h_{sh}	thickness of the conical shell

analysis of composite laminated shells are overviewed. Irie et al. [1] addressed natural frequency of isotropic truncated conical shell under different combinations of boundary conditions and cone angles. Lam and Hua investigated the effects of rotating velocity at different cone semi-vertex, circumferential wavenumber and boundary conditions on frequency parameter. In their work, Galerkin procedure was applied to analyze the problem. Jin et al. [2] assessed vibration behavior of truncated conical shells under general boundary conditions, geometrical properties and also modified Fourier series and Rayleigh–Ritz were applied to solve the problem. Based on FSDT and Hamilton principal, the governing equations of motion for rotating and non-rotating laminated composite conical shell under axially loading were derived by Daneshjou et al. [3]. GDQM and trigonometric functions towards circumferential direction were employed to obtained numerical results. Lam and Hua [4] investigated the influences of boundary conditions on the frequency parameter under different rotating velocities, cone angles and circumferential wavenumbers for rotating truncated conical shell using Galerkin Method. Tong [5] presented the free vibration of axisymmetric isotropic and cross-plied composite laminate conical shell to illustrate the effects of transverse shear deformations. Viswanathan et al. [6] carried out the free vibration analysis of angle-ply conical shell using FSDT theory. Bickley-type cubic spline and collocation procedure were used to obtain eigenvalue equation system and obtain natural frequency. Three-dimensional (3D) vibration analysis of hybrid laminated cylindrical shell embedded with piezoelectric layers has been presented by Alibeigloo and Kani [7]. 3D elasticity theory and differential quadrature state space method were utilized to derive and solve the governing equations, respectively. Nonlinear dynamic vibration analysis of laminated cylindrical shell was investigated by Dey and Ramachandra [8]. Donnell's shell theory combining FSDT were utilized to model the problem and also Galerkin's procedure and incremental Harmonic Balance method were employed to obtain numerical results. A set of characteristic orthogonal polynomials via the Gram–Schmidt process were applied to approximate displacement components. Moreover, Rayleigh and Fourier series methods were developed to obtain numerical results. Free vibration analysis of rotating cylindrical-conical shells was investigated by Sarkheil and Foumani [9]. It was assumed that displacement field is harmonic respect to time and circumferential direction. The power series method was employed to solve existing equations. Thin and Nguyen [10] proposed Dynamic Stiffness Method based on Reissner Mindlin theory for free vibration of fluid-filled composite circular cylindrical shell. They also used multi-vibration measuring machine to obtained experimental data. With aid of dynamic version of virtual work principle the governing equations were extracted. Navier double trigonometric method was applied to attain numerical results. Lei et al. [11] explored the geometrical nonlinear analysis of CNTR-FG composite perfectly bonded laminated plate. The meshless kernel particle function was used to approximate displacement components and stabilized nodal integration was applied to compute bending stiffness matrix. Ghayesh et al. [12] investigated the vibration and stability of an axially traveling laminated beam. Newton second law of motion and classical beam theory were utilized to obtain the governing equation of motion and also perturbation technique was employed to solve the equations. Biswal et al. [13] carried out experimental and numerical investigations of free vibration behavior of woven fiber Glass/Epoxy laminated composite shell. Zhao et al. [14] presented free and forced vibration analysis of laminated composite