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Heuristical and numerical considerations for the carbuncle phenomenon

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ABSTRACT

In this study, we investigate the so called carbuncle phenomenon by means of numerical experiments and heuristic considerations. We identify two main sources for the carbuncle: instability of the 1d shock position and low numerical viscosity on shear waves. We also describe how higher order stabilizes the 1d shock position and, thus, reduces the carbuncle.

1. Introduction

For the simulation of shock structures in multidimensional gas flows there are essentially two major approaches: shock fitting and shock capturing. The idea of the first class of schemes is to exactly detect the shock position and split the computational domain along the shock line into two (or for more complex structures even more) domains, leading to an almost perfect reproduction of shocks in the numerical solution [1–3]. Disadvantages of this method include the difficulty to deal with shocks which unexpectedly evolve in the domain and a practical restriction to certain shock structures. In order to overcome these restrictions, more and more scientists started to build their simulations on shock capturing schemes, which are designed to work without the knowledge of the exact shock position. Thus, the shock is captured in a (hopefully) thin layer of grid cells. For an overview over shock fitting schemes and some recent developments in that area, we refer to [4,5].

A main ingredient of shock capturing schemes are so called Riemann solvers: numerical (first order) flux functions, which are based on an approximate solution of the Riemann problem at the cell face. This class was later on expanded, starting with [6,7], through introduction of Flux-Vector-Splitting schemes, which are based on a splitting of the physical flux function, but in a wider sense can be considered as Riemann solvers. More important for our study, however, is the distinction between complete and incomplete Riemann solvers. While the first is designed to resolve all waves present in a Riemann problem, the latter will neglect some waves. Prominent examples of complete Riemann solvers are the schemes by Roe [8] and Osher [9]. While these solvers are preferable when complex wave structures as well as entropy and shear waves are expected, incomplete solvers are known to be robust in situations dominated by strong shocks. An example is the HLL-solver [10] whose construction is based only on the two outer waves of the Riemann problem. In the eighties and nineties, more and more applications were treated with above mentioned methods for gas dynamics. The methods were

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also extended to other hyperbolic conservation laws like shallow water or compressible magnetohydrodynamics (MHD). For a detailed discussion of shock capturing, we refer to the textbooks [11–17].

In the context of shock capturing, some irregularities were observed: properties of the discrete solutions which were by no means representations of physical phenomena. In gas dynamics simulations unphysical discrete shock structures and even a complete breakdown of the discrete shock profile could appear [18,19]. According to its form in blunt body problems, it was christened *carbuncle phenomenon*. Since the seminal paper of Quirk [19], an immense amount of research has been conducted on this instability problem. The origin of the name comes from the fact that in strongly supersonic flows against an infinite cylinder simulated on a body-fitted, structured mesh the middle part of the resulting bow shock degenerates to a carbuncle-shaped structure. It was conjectured already by Quirk [19] that this phenomenon is closely related to other instabilities such as the so-called *odd–even-decoupling* encountered in straight shocks aligned with the grid. Unfortunately, the failure is only found in schemes with high resolution of shear and entropy waves, so called complete Riemann solvers, which are needed to properly resolve the boundary layers and turbulent structures. This category includes for example the Godunov, Roe, Osher, HLLC and HLLEM schemes [11,20]. These schemes are preferable in calculations involving complex wave structures as well as boundary layers.

The research on the carbuncle was twofold. On the one hand, the stability of discrete shock profiles was investigated in one as well as in several space dimensions. On the other hand, a lot of effort was put into finding cures for the failure of some schemes in numerical calculations. For example, many cures that were offered are based on an indicator that tells the scheme when to switch to an incomplete Riemann solver. These indicators need information from other cell faces, making the numerical flux function non-local. It was found that even in one space dimension there are some instabilities of discrete shock profiles: slowly moving shocks produce small post-shock oscillations [19,21,22]. But also in the case of a steady shock, instabilities can be found depending on the value of the adiabatic coefficient γ as was shown by Bultelle et al. [23]. However, the connection to two-dimensional instabilities is still not fully understood.

The two-dimensional instabilities themselves seem to be closely related to each other. Chauvat et al. [24] show through an ingenious numerical investigation that the mechanisms driving the odd–even-decoupling and the carbuncle are closely related. Dumbser et al. [25] present a method to test Riemann solvers for their tendency to odd–even-decoupling. Here, the basic idea is to discretize a steady shock in space and test the linear stability of the system of ODEs resulting from the Method of Lines. This allows for all tested solvers to predict whether they would evolve an instability or not. There is also a number of experimental studies of the carbuncle, especially the influence of the underlying numerical flux function [26–29], with the goal to identify the "optimal Riemann solver". Finally, Elling [30] found a connection to physical shock instabilities.

Most these investigations have in common that they (a) intend to find a single source for the carbuncle, (b) do not take into account the influence of the order of the scheme—they usually compare different Riemann solvers in a scheme with fixed order—, and (c) do not distinguish between the contribution of entropy or shear waves to the carbuncle. The most surprising is case (b) since it is well known that in higher order schemes the carbuncle is much weaker than in first order; for very high orders, it is essentially absent. The purpose of this paper is to fill these gaps in research. We want to study the influence of the (1d) stability of the shock position and the 2d or 3d features such as vorticity separately. In this course, we also try to separate the influences of entropy and shear waves. But the main focus (and main novelty) of this study is that we investigate the influence of the order of the scheme on the stability of the (1d) shock position. We will show how increasing the order of the scheme, despite of lowering the numerical shear viscosity, stabilizes the 1d shock position.

The outline of the paper is as follows: In Section 2 we give a short representative review of some theoretical results. The insight gained by these results provides us with the guidelines for our numerical experiments. In Section 3, we give a review of the schemes we use in our numerical experiments. The numerical test cases are introduced in Section 4. The main results are presented in Sections 5 (one-dimensional issues) and Section 6 (multi-dimensional issues), followed by some conclusions and directions for further research in Section 7.

2. Short review of the theory

There are many papers dedicated to the carbuncle phenomenon [19,21,23,24,30–40], however only few of them discuss the origins of the carbuncle from a theoretical point of view. Here, we give a short representative review of some theoretical results.

2.1. Contribution by Bultelle et al.

Bultelle et al. [23] investigate steady shocks in one space dimension. A first study of these was done by Barth [41] who found that for a perfect gas with $\gamma = 1.4$, flux functions which enforce the Rankine–Hugoniot condition at discontinuities may have transition states which are unstable to perturbations when the preshock Mach number is greater than six. Bultelle et al. go even further. They prove that the Godunov scheme for strong steady shocks and an adiabatic coefficient $1 \leq \gamma^* \approx 1.62673$, with γ^* being a root of

$$\gamma^4 + 3\gamma^3 - 21\gamma^2 + 17\gamma + 8 = 0,$$

can produce unstable shock profiles. They report that in practice, after a transient regime, the unstable leads to a stable profile with the intermediate state in a neighboring cell or even one cell further. If in neighboring grid slices normal to the