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Mathematical model to investigate the behaviour of the systems of ferromagnetic particles under the magnetic fields

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A R T I C L E I N F O

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A B S T R A C T

In this work, a comprehensive mathematical analysis is presented to explore the behaviour of multi-sphere system of ferromagnetic particles in a homogeneous medium under the uniform magnetic fields. We assume that the geometrical shapes of the particles are spheres. The magnetic field intensity due to all particles and the external field is obtained by the superposition of the magnetic potentials in the system. For that, the translational addition theorems were used to express the functions in the coordinates system attached to a specific particle. Further, by imposing the exact boundary conditions, the field quantities outside the particles are solved. Then, these known quantities and the boundary conditions are used to obtain the field quantities inside the particles in the system. Finally, from the derived expressions, we generate benchmark accurate numerical results for various values of the characteristic parameters such as the radii of the particles and the relative distance between the particles, at the points outside as well as inside of a system of three ferromagnetic particles in the presence of an uniform magnetic field. The generated numerical results are analysed qualitatively and quantitatively, and are validated by using theoretical concepts related to the magnetic field on ferromagnetic spheres with some specific geometric configurations involved.

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1. Introduction

The investigation of the behaviour of ferromagnetic particles in an external magnetic field is important for numerous range of applications in magnetostatics boundary value problems from biomedical science to engineering applications [1]. For instances, in [2], the authors have considered the behaviour of ferromagnetic nano-particles in and around the blood vessels in the presence of magnetic field. They have compared the results with previously published experiments. Furthermore, a mathematical model for capturing magnetic nano-particles for magnetic drug targeting, is investigated in [3]. On the other hand, in nano-scale physics [4], fluid mechanics [5], and ferrohydrodynamic [6] are some of the areas that widely use ferromagnetic bodies.

In ferromagnetic materials, each atom has a relatively large dipole moment, and inter-atomic forces cause these moments to array in a parallel orientations over regions containing a large number of atoms which each have a strong magnetic moment with vary in direction from region to region as shown in Fig. 1(a). Therefore, the overall effect is cancelled out

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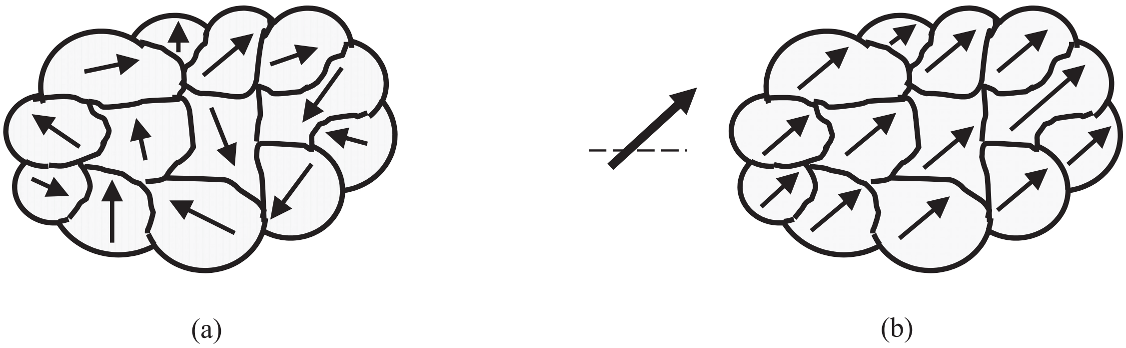

Fig. 1. Ferromagnetic material in (a) random region orientations (b) a magnetisation along the direction of the external field.

and the material has no magnetic moment. But, in the presence of an external magnetic field (see Fig. 1(b)), those regions which have moments in the direction of the applied field increase the sizes (moments) of their neighbours. Thus, the magnetic field inside (magnetisation) increases greatly along the direction of the external field [7].

In this study, we assume the geometrical shapes of all particles are spheres, with an ideal ferromagnetic materials. As a result of that, the material has the characteristics of linear magnetic and infinite permeability. Then, the magnetic field lines are perpendicular (due to the assumption we made, shapes of the particles are spheres) to the surface of the particles. Therefore, the scalar magnetic potential at the surface is always constant. The magnetic field intensity due to all particles and the external field is obtained by the superposition of the magnetic potentials in the system.

In order to impose the boundary conditions at the surface of the particles, it is necessary to obtain the field quantities (due to all particles and the external field) in terms of the coordinates of the system attached to one particle. For that, the translational addition theorems [8] were used to express the functions in the coordinates system attached to a specific particle. Then, by imposing the exact boundary conditions, the magnetic field intensity outside the particles are solved. Next, these known quantities and the boundary conditions are used to obtain the magnetic flux density inside the particles in the system. Finally, from the derived expressions, we generate benchmark accurate numerical results for various values of the characteristic parameters such as the relative particle sizes and the relative distance between the particles, at points outside as well as inside of a system of three ferromagnetic particles in the presence of an external magnetic field.

This paper is organised as follows. In Section 2, we introduce the principle equations related to the theories and for the boundary conditions that use in this study. A mathematical model for multi-sphere ferromagnetic particles is obtained in Section 3. In addition, as an example, the behaviour of a system of three ferromagnetic spheres is also investigated in the section. Section 4 shows the generated numerical results for various field quantities, and is analysed the results qualitatively and quantitatively for controllable accuracy. Finally, the summary and the conclusions are stated in Section 5.

2. Governing equations and boundary conditions

In this section, we consider the corresponding mathematical equations for the main theories used in this study, namely, the solution to the Laplace equation in spherical coordinates, the translational addition theorems for spherical Laplacian functions, and the boundary conditions on the magnetic fields.

2.1. Solution to the Laplace equation in spherical coordinates

Magnetic fields are classically described by Maxwell's equations [9]. Since this research is on static magnetic fields, we specialise to the case of magnetostatic equations. Whenever the current density is zero, starting from Ampère's circuital law of Maxwell's equations [10], we can easily prove the magnetic field intensity $\mathbf{H}$ as [11]

$$\mathbf{H} = -\nabla f \tag{1}$$

where f is the scalar magnetic potential.

In spherical coordinates (r, θ, φ), the general solution of the Laplace equation

$$\nabla^2 f = \left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right] f = 0 \tag{2}$$

can be expressed in the form [12]

$$f = \sum_{n=0}^{\infty}\sum_{m=-n}^{n}(C_{nm}r^{-(n+1)} + D_{nm}r^n)P_n^m(\cos\theta)\,e^{-jm\varphi} \tag{3}$$

where C_{nm} and D_{nm} are constants of integration, and m and n are integers. P_n^m are the associated Legendre functions of first kind, of degree n and order m [13].

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