



Optimal harvesting strategy based on rearrangements of functions



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ABSTRACT

We study the problem of optimal harvesting of a marine species in a bounded domain, with the aim of minimizing harm to the species, under the general assumption that the fishing boats have different capacities. This is a generalization of a result of Kurata and Shi, in which the boats were assumed to have the same maximum harvesting capacity. For this generalization, we need a completely different approach. As such, we use the theory of rearrangements of functions. We prove existence of solutions, and obtain an optimality condition which indicates that the more aggressive harvesting must be pushed toward the boundary of the domain. Furthermore, we prove that radial and Steiner symmetries of the domain are preserved by the solutions. We will also devise an algorithm for numerical solution of the problem, and present the results of some numerical experiments.

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1. Introduction

Kurata and Shi [1] considered the problem of optimal harvesting of a marine species with the aim of minimizing harm to the species. From an applied marine economy perspective, the main result of their work is a formal argument in support of designation of no-harvesting zones.

To be more precise, consider a species of (say) fish living in a lake Ω with boundary $\partial\Omega$. The species is a source of income for fishermen, and as such, it is imperative to come up with harvesting strategies that, on the one hand, allow the fishermen to make a living, and on the other hand, minimize harm to the species and provide long term sustainability of the source of income.

In mathematical biology, reaction-diffusion equations have provided a standard model for the study of population dynamics of many species. Assuming that w denotes the population density, a fairly general form of a reaction-diffusion equation may be expressed as:

$$\partial w / \partial t = \nabla \cdot (D \nabla w) + f(w, x, t), \quad x \in \Omega, t > 0, \quad (1)$$

in which f is regarded as the source function [2, Chap. 11]. For modelling the fish population in a lake, we consider the same set of assumptions as taken by Oruganti et al. [3], i. e.,:

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1. The fish move around via a 'random walk' in the bounded *homogeneous* environment Ω , which implies that D is constant throughout the domain. Of course, in a more general setting, it could be a function of w , x , and t .
2. The population dynamics is subject to logistic growth.

With these assumptions, one obtains the following equation:

$$\partial w/\partial t = D\Delta w + aw(1 - w/K), \quad x \in \Omega, t > 0, \quad (2)$$

where $D > 0$, and in which $a, K > 0$ denote the linear reproduction rate and the carrying capacity of the environment, respectively. If the species is subject to harvesting, then a further term should be subtracted from the right hand side of (2) to account for the loss of population due to harvesting. Hence, the modified equation takes the form:

$$\partial w/\partial t = D\Delta w + aw(1 - w/K) - h(x, w), \quad x \in \Omega, t > 0, \quad (3)$$

in which $h(x, w)$ denotes the harvesting density per unit time.

As we are interested in qualitative properties of solutions, we would like to make some simplifications in order to turn (3) into a more manageable one. First, we assume that the harvesting density per unit time is directly proportional to the density w and some harvesting effort $E(x)$ at each point $x \in \Omega$. Hence, we take h to be of the form:

$$h(x, w) = E(x)w.$$

This is acceptable as, for instance, fishers would cast the same type of net at a point $x \in \Omega$ regardless of the density of population at that point.

Second, to avoid clutter, we would like to take the specific values that a and K take out of the discussion. To that end, we introduce the change of variables $w(x, t) = Ku(y, t)$ and $y = \sqrt{a}x$ into (3), which leads to:

$$\partial u/\partial t = a(D\Delta u + u(1 - u)) - E(y/\sqrt{a})u, \quad y \in \Omega', t > 0.$$

Indeed, for qualitative analysis of the solutions, the parameter a appearing in the right hand side has no significance in what shall follow (see, e. g., [4]). Hence, we normalize the value of a to 1. Combining the effects of all the simplifications, and by taking the diffusion scale ϵ to be $\epsilon := \sqrt{D}$, we obtain:

$$\partial u/\partial t = \epsilon^2 \Delta u + u - u^2 - E(y)u, \quad y \in \Omega', t > 0.$$

Regarding boundary conditions, we assume that the surrounding area of the region Ω is completely inhospitable, e. g., Ω is a lake surrounded by land. This leads to the choice of Dirichlet boundary conditions. We remark here that a more general problem may be studied by considering Robin boundary conditions, but as, on the one hand, our focus in this article is not the effects of the boundary conditions, and on the other hand, with Dirichlet boundary conditions we have richer mathematical results at our disposal, we focus on the case of Dirichlet boundary conditions.

Putting together all the normalizations, simplifications, and considerations alluded to thus far leads to the dynamics of the fish population being modeled by the following reaction-diffusion equation with logistic growth:

$$\begin{cases} \partial u/\partial t = \epsilon^2 \Delta u + u - u^2 - E(x)u, & x \in \Omega, t > 0, \\ u(x, t) = 0, & x \in \partial\Omega, t > 0, \\ u(x, 0) = u_0(x), & x \in \Omega, \end{cases} \quad (4)$$

in which:

- $\Omega \subseteq \mathbb{R}^N$ is a smooth domain, with $N \geq 1$;
- u is the population density;
- $u_0 \geq 0$ is an initial population density;
- ϵ is the diffusion scale;
- $E(x) \geq 0$ is the harvesting effort, and as a consequence, $E(x)u$ is the harvesting density per unit time;
- All the variables are dimensionless.

The *biological energy function* associated with system (4) is given as follows:

$$\mathcal{E}_\epsilon(u, E) := \underbrace{\frac{\epsilon^2}{2} \int_\Omega |\nabla u|^2 dx}_{\text{kinetic energy}} - \underbrace{\frac{1}{2} \int_\Omega u^2 dx + \frac{1}{3} \int_\Omega u^3 dx + \frac{1}{2} \int_\Omega E(x)u^2 dx}_{\text{potential energy}}, \quad (5)$$

for $u \in H_0^1(\Omega)$. As time passes, this quantity decreases for $u(\cdot, t)$. To see this, observe that using the Divergence theorem,

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