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New insight into reachable set estimation for uncertain singular time-delay systems

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ABSTRACT

This paper investigates the problem of reachable set estimation for a class of uncertain singular systems with time-varying delays from a new point of view. Our consideration is centered on the design of a proportional-derivative state feedback controller (PDSFC) such that the considered singular system is robustly normalizable and all the states of the closed-loop system can be contained by a bounded set under zero initial conditions. First, a nominal singular time-delay system is considered and sufficient conditions are obtained in terms of matrix inequalities for the existence of a PDSFC and an ellipsoid. In this case, the considered system is guaranteed to be normalizable and the reachable set of the closed-loop systems is contained by the ellipsoid. Then, the result is extended to the case of singular time-delay systems with polytopic uncertainties and relaxed conditions are derived by introducing some weighting matrix variables. Furthermore, based on the obtained results, the reachable set of the considered closed-loop singular system can be contained in a prescribed ellipsoid. Finally, the effectiveness of our results are demonstrated by two numerical examples.

1. Introduction

During the past years, reachable set estimation has been widely applied in control systems with actuator saturation, peak-to-peak gain minimization, and parameter estimation [1,2]. The objective of reachable set estimation is to find a suitable region which can bound all the reachable states of a dynamic system with input disturbances and zero initial conditions. Recently, many researchers have been devoting to studying the problem of reachable set estimation for time-delay systems since time delays often exist in physical systems and may affect the stability and performance of systems. For example, to obtain the existence conditions of an ellipsoid which can bound all the reachable states of linear systems with time-varying delays, Lyapunov–Krasovskii functionals (LKFs) were used in [3–6]; these conditions are less conservative than those in [2] by using Lyapunov–Razumikhin functions. To further improve the conditions of reachable set estimation, a non-uniform delay-partitioning method and a triple integral technique were used in [5]; in [6], a relaxed LKF was constructed where all the involved symmetric matrices are not required to be positive definite. For the problem of reachable set estimation for linear systems with distributed delays; see, e.g., [7–9] and the reference therein.

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On the other hand, singular time-delay systems have also received great attention and lots of results have been developed in the literatures for the analysis and synthesis of continuous- or discrete-time singular systems with time delays, such as admissibility analysis and stabilization [10–13], H_∞ or passive control [14–17], dissipativity analysis [18–20], sliding mode control [21,22], and so on. It is noted that some results on reachable set estimation for singular time-delay systems can also be found; see, e.g. [23,24]. The methods of dealing with the problem of reachable set estimation for normal time-delay systems can only guarantee that the reachable states of the slow subsystem of a singular system are bounded by an ellipsoid. In [23], different bounding techniques were used for the slow subsystem and the fast subsystem of a considered singular time-delay system, respectively. All the reachable states of the considered singular system were contained by an intersection of ellipsoids. However, to decompose the considered singular system into two subsystems with special structures, two nonsingular matrices were required to be found in [23], in which the obtained bounding ellipsoids were related to the two nonsingular matrices. This increases the difficulty of seeking the "smallest" ellipsoid, especially when the structures of the considered systems are complicated. Besides, the applications of the result in [23] are limited since an assumption was introduced. Therefore, the study on the reachable set estimation for singular time-delay systems is still challenging.

In this paper, we present a new insight into the reachable set estimation for a class of uncertain singular time-delay systems, where the considered singular system does not need to be decomposed and a proportional-derivative state feedback controller (PDSFC) is designed to guarantee all the reachable states of the considered closed-loop singular system contained by an ellipsoid. It is worth mentioning that though PDSFCs have been widely used in singular systems; see, e.g. [25–30], we first use them to study the problem of reachable set estimation for uncertain singular systems with time-varying delays. The main contributions are stated as follows.

(i) By using a PDSFC, the considered (uncertain) singular system is (robustly) normalizable and all the reachable states of the closed-loop system are contained by an ellipsoid. By an optimization problem, the "smallest" ellipsoid can be obtained easily. Since the considered (uncertain) singular system does not need to be decomposed and no assumptions are given, our results are less conservative.
(ii) The problem of reachable set estimation for singular time-delay systems with polytopic uncertainties is also studied. The uncertainties exist in both the state matrix and the derivative matrix. By introducing some weighting matrix variables, relaxed conditions are presented.
(iii) Based on the obtained conditions, we can investigate two problems by designing a PDSFC: One is to find the "smallest" ellipsoid to contain all the reachable states of the considered closed-loop singular system; the other is to make the reachable set of the considered closed-loop singular system contained in a prescribed ellipsoid.

The rest part of the paper is organized as follows. Section 2 is the problem formulation which gives the system model and some useful lemmas. Section 3 is the main results. A PDSFC is designed in this section such that the considered nominal singular time-delay system is normalizable and all the reachable states of the closed-loop system are contained by an ellipsoid. Then the result is extended to the case of singular time-delay systems with polytopic uncertainties. Section 4 gives two numerical examples to illustrate the effectiveness of our results and Section 5 is the conclusion of the paper.

Notations: Throughout this paper, $\mathbf{R}^n$ denotes the n-dimensional Euclidean space over the reals and $\mathbf{R}^{n\times m}$ denotes the set of all $n\times m$ real matrices. $|\cdot|$ denotes the Euclidean norm for vectors and $\|\cdot\|$ denotes the spectral norm for matrices. For real symmetric matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite). I is an identity matrix with appropriate dimension and 0 is a zero matrix with appropriate dimension. The superscripts 'T' and '-1' represent the matrix transpose and inverse, respectively. $\star$ denotes the symmetric term in a symmetric matrix; $\rho(M)$ denotes the spectral radius of the matrix M; $rank(\cdot)$ stands for the rank of a matrix; $det(\cdot)$ denotes the determinant of a square matrix with appropriate dimension. For any square matrices A and B, define $diag(A, B) = \begin{bmatrix} A & 0 \\ \star & B \end{bmatrix}$. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Problem formulation

Consider the following uncertain singular system with a time-varying delay:

$$\begin{aligned} &E(\lambda)\dot{x}(t) = A(\lambda)x(t) + A_d(\lambda)x(t-d(t)) + B(\lambda)u(t) + D(\lambda)\omega(t), \\ &\qquad x(t) \equiv 0, \quad \forall t \in [-d_2, 0], \end{aligned} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state vector; $u(t) \in \mathbf{R}^m$ is the control input; the matrix $E(\lambda) \in \mathbf{R}^{n\times n}$ may be singular and it is assumed that $rank(E(\lambda)) = r \leq n$; $d(t)$ is a time-varying continuous function that satisfies

$$0 \leq d_1 \leq d(t) \leq d_2, \qquad \dot{d}(t) \leq \mu < \infty, \tag{2}$$

where d_1, d_2, μ are constants; $\omega(t) \in \mathbf{R}^l$ is the disturbance satisfying

$$\omega^T(t)\omega(t) \leq \bar{\omega}^2, \tag{3}$$

where $\bar{\omega}$ is a real constant.