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Trajectories reconstruction of spinning baseball pitches by three-point-based algorithm

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A B S T R A C T

In this work, a method to reconstruct trajectories by obtaining the initial conditions, velocity and angular velocity, from spinning throws is designed. It is based on considering Magnus effect can be separated from rest of forces that define the dynamics of the ball, such assumption is supported by an energetic analysis. Thus, the methodology consists in solving the two-point boundary value problem (BVP) of the movement equations without Magnus force and then adding its effect.

We found that only three points (ball positions as functions of time) of a trajectory are necessary to characterize it, and consequently to solve the three-point BVP. This result is applied in an algorithm based on the shooting method, which obtains the initial conditions of synthetic trajectories by minimizing the distance between points of the original trajectory and those of the proposed trajectory, with high accuracy in a short time.

1. Introduction

1.1. Background and summary

In the context of baseball, the reconstruction of trajectories is generally applied to replay of pitches on broadcast of games. Existing methods, focus only on tracking the ball by means of tools processing all image available. They consist of three basic steps: First, an overlap of a lot of photos of the same pitch is carried out to obtain the position of the ball at different times; also it is used to transform the 3D reality into a 2D image. Then, several trajectories are proposed using probabilistic methods, database comparisons, and/or color and region filtering based on pixel analyses [1–5]. Thus, some discrete trajectories are filled with the same methods or by a parameter estimation [6]. Lastly, chosen trajectories are generally compared with mathematical models of the ball dynamics.

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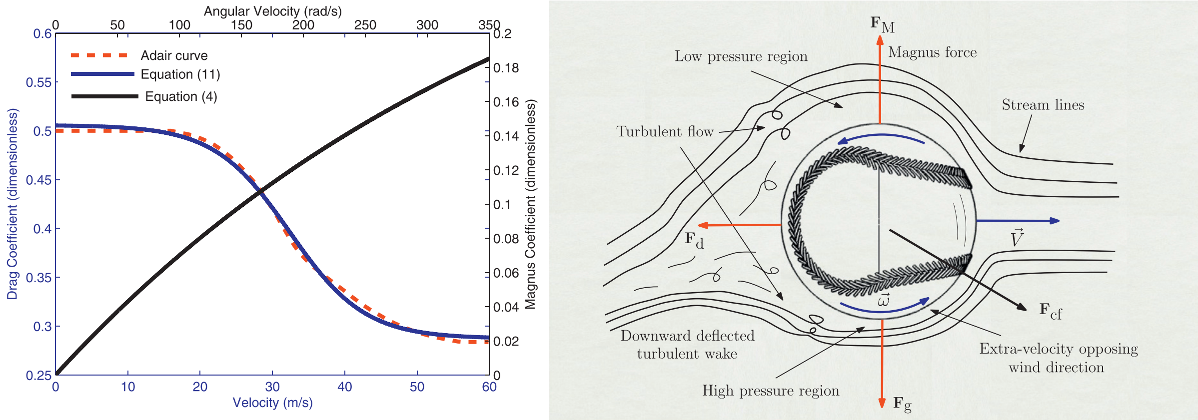

Fig. 1. Left: Drag (C_d) and Magnus (C_M) coefficients. Red dashed line represents the Adair estimation (data were extracted using tools of Matlab R2013a program) for C_d [15]. Blue solid line approximates Adair curve (Eq. (11)), while black line fits to Briggs experimental data [29] using Eq. (4). Right: Diagram of forces and stream lines of a rotating baseball moving to the right with an angular velocity $\boldsymbol{\omega}$. The forces involved are gravitational ($\mathbf{F}_g$), drag ($\mathbf{F}_d$) and Magnus ($\mathbf{F}_M$). Centrifugal force ($\mathbf{F}_{cf}$) is also included in. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

In this way, such methods usually do not deal with the movement equations of the ball so that a large number of ball positions data is required only to complete the trajectory but initial conditions of the ball are not obtained. However, they have been reported feasible and bringing good results for tracking the ball. On the other hand, the methodologies involving to solve the BVP of the ball movement equations are poorly viable for their use because of the data or the equipment required [7,8].

We designed a method to reconstruct trajectories by obtaining the initial conditions of spinning throws taking into account the movement equations of the ball mass center. The work is focused on pitches with initial conditions, velocity $\mathbf{V} \equiv (V_x, V_y, V_z)$ and angular velocity $\boldsymbol{\omega} \equiv (\omega_x, \omega_y, \omega_z)$, around the range of a throw made by a professional pitcher, namely, $\mathbf{V} \in [(-3, 30, -3), (3, 50, 3)]$ m/s, $|\boldsymbol{\omega}| \in [100, 310]$ rad/s [9], fixing y-axis in mound-home direction, z-axis perpendicular to the Earth's surface, and x-axis orthogonal to both y and z axes, according to the right hand rule.

This paper is structured as follows: Section 1.2 has the purpose of presenting the equations of motion of the ball. Basis of the method is to consider Magnus force can be separated from rest of forces that define the dynamics of the ball; we prove such assumption in Section 2.1 by an energetic analysis, and estimate the Magnus effect in Section 2.2 numerically. In Section 3, we describe the method (Section 3.1) and present an algorithm (Section 3.2). Results and conclusions are shown in Sections 4 and 5, respectively.

1.2. Dynamics of spinning pitches

Dynamics of a rotating baseball with mass m and moving at a velocity $\mathbf{V}$ is defined by the Newton's second law [10,11], such that

$$m\dot{\mathbf{V}} = \mathbf{F}_B + \oint_S \sigma \, dS \cdot \hat{\mathbf{n}}, \tag{1}$$

where the first term on the right side is the sum of forces acting through the volume of the ball (gravitational and Coriolis forces, and pseudo forces); whereas the second term is the net force, where its components are defined by the stress tensor σ, acting across the surface S of the ball, such it covers all forces produced as a consequence of the interaction air-ball: drag, Magnus, torque, cross and constraint forces [11].

When a ball is thrown with high spins, like those inside the range mentioned in Section 1.1, the forces caused by the asymmetries of the ball seams, cross forces, vary from positive to negative quickly and have sufficiently low magnitude to be omitted [12,13]. In addition, low decelerations on angular frequency ω – around 0.08% for curveballs [14] and 20% for golf balls after a flight of 5 s – together with the low precession of the spin axis $\hat{\boldsymbol{\omega}}$ let not consider torque forces. Moreover, constraint forces are commonly omitted because the stiffness of the ball is practically not affected when it travels through the air [15].

Thus, main forces involved in the integral of the right side of the Eq. (1) are drag and Magnus. Drag can be explained by the difference of pressure between front and back sides of the ball [16]. As shown in Fig. 1, stream lines on the back side of the ball are farther than on the front. This creates a low pressure region on the back side that causes a momentum with reverse direction to its motion. Experimental data suggest that difference of pressure and drag force are connected by an approximation proportional to the square of the ball speed [15,17,18], so that

$$F_d = \frac{1}{2}\rho A C_d V^2 \tag{2}$$