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Calculations of power system electromechanical eigenvalues based on analysis of instantaneous power waveforms at different disturbances

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ABSTRACT

In the paper, there are presented the calculation results of eigenvalues associated with electromechanical phenomena (i.e. electromechanical eigenvalues) of the power system (PS) model state matrix based on the analysis of instantaneous power disturbance waveforms. The method used for calculations does not require determining the PS (linearized in the operating point) state matrix of large dimensions. The disturbances taken into account in the investigations are: pulse or step changes (with a small amplitude) in the voltage regulator reference voltage in one of generating units, a symmetrical short-circuit in a transmission line and a change in the value of the PS line equivalent impedance (due to, e.g., disconnecting or connecting a large power load). In the PS model, there was taken into account the impact of a central frequency regulator. The presented method for calculations of eigenvalues consists of approximation of the instantaneous power waveforms by the superposition of modal components associated with the searched eigenvalues. This approximation can be performed by minimization of the objective function defined as a mean square error between the approximated and approximating waveform. The hybrid optimization algorithm, being a serial combination of genetic and gradient algorithms, was used for minimization of the so-defined objective function. Such a combination allows eliminating the main disadvantages of both component algorithms. In order to avoid calculation errors caused by the algorithm freezing in an objective function local minimum, the calculations of eigenvalues were carried out repeatedly for each disturbance waveform. The calculation results with the objective function values larger than the assumed limit value were rejected. The arithmetic means from the non-rejected calculation results were assumed to be the final calculation results of the real and imaginary parts of particular eigenvalues.

1. Introduction

The power system (PS) works properly, ensuring the supply of electric power to customers, only when its angular stability is maintained. A loss of its angular stability may be a reason of a serious system failure that results in a loss of power supply to a large number of customers. The power system angular stability can be assessed using angular stability factors [1,2,3] calculated on the basis of the system state matrix eigenvalues associated with electromechanical phenomena [1]. Further in the paper, they are called “electromechanical eigenvalues”.

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These eigenvalues can be calculated based on the state matrix of the power system mathematical model linearized around a steady state operating point. In this case the calculation results depend indirectly on the assumed models of particular power system components and their parameters. The model parameters used in calculations are often not sufficiently accurate and reliable [2–5].

The eigenvalues can also be calculated with a good accuracy on the basis of the analysis of actual disturbance waveforms appearing in the system after various disturbances [2,3,4,6]. In this case, calculation results are not affected by the assumed power system model and its parameters, but only by the current system performance [2,3,4].

Calculation of the power system state matrix eigenvalues based on the analysis of transient waveforms of selected quantities is called modal analysis. There are two types of this analysis [3,7]:

- *Experimental modal analysis*—there are taken into account the disturbance waveforms occurring after the purposeful introduction of a test disturbance to the system. The term “experimental” is meant to emphasize the fact that there is carried out an experiment consisting of introducing a disturbance.
- *Operational modal analysis*—consists of using measurements during PS operation without introducing a test disturbance. Dynamic waveforms appear under the influence of stochastic disturbances, e.g. short-circuits or stochastic load power changes.

The aim of the investigations was to calculate the electromechanical eigenvalues on the basis of instantaneous power disturbance waveforms of generating units occurring after:

- a rectangular pulse or step change in the voltage regulator reference voltage in one of generating units,
- a symmetrical low-current short-circuit in a transmission line,
- a change in the value of the PS line equivalent impedance (due to, e.g., disconnecting or connecting a large power load).

It was assumed that the angular stability of the PS was maintained after introducing the disturbance.

2. The linearized power system model

The power system model linearized around the steady working point is described by the state equation and output equations [2–4]:

$$\Delta\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\Delta\boldsymbol{x}(t) + \boldsymbol{B}\Delta\boldsymbol{u}(t)\,, \tag{1}$$

$$\Delta\boldsymbol{y}(t) = \boldsymbol{C}\Delta\boldsymbol{x}(t) + \boldsymbol{D}\Delta\boldsymbol{u}(t)\,, \tag{2}$$

where: $\Delta\boldsymbol{x}(t)$, $\Delta\boldsymbol{u}(t)$, $\Delta\boldsymbol{y}(t)$—vectors of the deviations from the steady values of, respectively: state variables, input variables and output variables, $\boldsymbol{A}$—state matrix. Matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$ from the state Eq. (1) and output Eq. (2) are calculated for the steady working point of the PS [2,3,4]. There were assumed the following dimensions of the vectors and matrices from Eqs. (1) and (2): $\Delta\boldsymbol{x} - n$, $\Delta\boldsymbol{u} - k$, $\Delta\boldsymbol{y}(t) - l$, $\boldsymbol{A} - n \times n$, $\boldsymbol{B} - n \times k$, $\boldsymbol{C} - l \times n$, $\boldsymbol{D} - l \times k$.

2.1. Solution of the state and output equations of the PS

The waveforms of output quantities of the linearized PS model can be calculated directly by integrating state Eq. (1) with the use of one of numerical integration methods. These methods allow obtaining a solution in the form of series of numerical values of individual state variables and output variables at specific time instants. However, numerical integration of the state and output equations does not allow a direct qualitative analysis of the obtained results [3].

Differential Eq. (1) can also be solved analytically. The solutions of heterogeneous state Eq. (1) and output Eq. (2) are of the following form (when assuming the values of the vector $\Delta\boldsymbol{x}(t_0) = \boldsymbol{0}$ for the initial instant of analysis t_0) [3]:

$$\Delta\boldsymbol{x}(t) = \int_{t_0}^{t} \exp(\boldsymbol{A}(t-\tau))\,\boldsymbol{B}\Delta\boldsymbol{u}(\tau)\,\mathrm{d}\tau, \tag{3}$$

$$\Delta\boldsymbol{y}(t) = \int_{t_0}^{t} \boldsymbol{C}\exp(\boldsymbol{A}(t-\tau))\,\boldsymbol{B}\Delta\boldsymbol{u}(\tau)\,\mathrm{d}\tau + \boldsymbol{D}\Delta\boldsymbol{u}(t)\,. \tag{4}$$

The matrix $\exp(\boldsymbol{A}t)$ is the square matrix of the same size $n \times n$ as the matrix $\boldsymbol{A}$. There are many methods of calculating the value $\exp(\boldsymbol{A}t)$, among others using a Taylor series or the Jordan canonical form of the matrix $\boldsymbol{A}$ [3,7,8,9]. In this paper, there will be used the Jordan canonical form of the matrix $\boldsymbol{A}$ obtained by means of eigenvalues and eigenvectors of the matrix $\boldsymbol{A}$ [3,7,8,9].

If all the eigenvalues of the matrix $\boldsymbol{A}$ are singular, which is practically always true for PS models, one can assign right- $\boldsymbol{V}_h$ and left-side $\boldsymbol{W}_h$ column eigenvectors of size n to each eigenvalue $\lambda_h = \alpha_h + \mathrm{j}\nu_h$ (for $h = 1, 2, \ldots, n$). Then in calculations, there are often used the right- $\boldsymbol{V}$ and left-side $\boldsymbol{W}$ modal matrices of the size $n \times n$ whose columns contain successive right- and left-side eigenvectors, respectively, as well as the diagonal matrix $\boldsymbol{\Lambda}$ of the size $n \times n$ whose main diagonal contains