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Investigation of generalization ability of a trained stochastic kinetic model of neuron

Aleksandra Świetlicka [a,*], Krzysztof Kolanowski [a], Rafał Kapela [a], Mirosław Galicki [b], Andrzej Rybarczyk [a]

[a] Chair of Computer Engineering, Poznan University of Technology, ul. Piotrowo 3A, 60-965 Poznań, Poland
[b] Department of Mechanical Engineering, University of Zielona Góra, ul. Prof. Szafrana 4, 65-516 Zielona Góra, Poland

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A B S T R A C T

In this work we focus on the generalization ability of a biological neuron model. We consider a Hodgkin–Huxley type of biological neuron model, based on Markov kinetic schemes, trained with the gradient descent algorithm.

The examination of the generalization ability of the kinetic model of neuron is performed with methods derived from the regularization theory. The error function of the neuron model is supplemented with a regularizer. We examine two different forms of the regularizer: a penalty function, which is a sum of squared weights of the neuron model, and a Tikhonov functional, which is a linear differential operator related to the input–output mapping.

As an example we consider a stochastic kinetic model of neuron to solve a problem of noise reduction in an image.

Additionally in the paper we present different measures to show that adding a regularizer to the error function does not worsen the obtained results of noise reduction in an image.

1. Introduction

Nowadays biological neural networks are the tool of choice for solving numerous computational problems. Classification and identification tasks and control of dynamic processes are commonly performed with this kind of tool. That is why it is so important to invest in further research in this area and focus not only on applications but also on improving abilities of biological neural networks.

A general problem for a neural network is to find a proper input–output mapping based on a learning set of input and output vectors. It is well known that this task is ill posed [1], as its solution is not unique. It is necessary, then, to apply some adequate criterion, which would allow for a test of neural network's performance. The natural criterion is the generalization ability, as it is one of the basic features of a neural network in general. A valuable overview of generalization ability of neural networks can be found e.g. in [2].

* Corresponding author.
E-mail addresses: aleksandra.swietlicka@put.poznan.pl (A. Świetlicka), krzysztof.kolanowski@put.poznan.pl (K. Kolanowski), rafal.kapela@put.poznan.pl (R. Kapela), m.galicki@ibem.uz.zgora.pl (M. Galicki), andrzej.rybarczyk@put.poznan.pl (A. Rybarczyk).

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Table 1
Parameters of Hodgkin–Huxley model.

i	V_i [mV]	g_i [ms/cm^2]
Na	115	120
K	−12	36
L	10.6	0.3

The generalization ability determines whether a neural network is capable of returning a proper solution for data that did not appear in the training set. Moreover, ability of generalization is important, as it prevents the training algorithm from over-training the neural network.

There are many methods of analyzing a neural network's generalization ability. A detailed description of some of these methods can be found in [3]. One of the simplest methods is to examine the Vapnik–Chervonenkis dimension [4], which, unfortunately, requires the knowledge of a sufficiently large training set [2]. Another technique is based on methods incorporated in the regularization theory. In this technique a regularizer is added to the neural network's error function [5]. Depending on the definition of the regularizer, this additional component of the error function can penalize the curvature of the neural network [6] or stabilize the solution, making it smooth [5].

In this paper we focus on the biological neuron model, also known as spiking neuron model, which provides information about electrical properties of the neuron. The neuron model can be included in the group of neural networks, as a special case. Belonging to the group of neural networks imposes higher requirements on biological neuron models to improve features that, in general, neural networks have, and consequently can be applied in practical problems.

We examine the stochastic kinetic model of neuron, which is an extension of the Hodgkin–Huxley model. While generalization ability is one of the most important properties of neural networks, it is necessary to check if this kind of spiking neuron model has this feature. Investigation of generalization ability of stochastic kinetic model of neuron is performed by supplementing the error function of the model with a regularizer. It is important to, first, check whether the regularizer does not worsen the spiking neuron model's performance and, next, if it can improve this performance.

The purpose of this work is to analyze generalization ability of the stochastic kinetic model of neuron. To perform this investigation we have used two different forms of regularizer, penalty function and generalization term (also called Tikhonov functional). The penalty function is the sum of squared weights of the model [6], while the Tikhonov functional is a linear differential operator related to input–output mapping [5]. The model of neuron was trained with the gradient descent algorithm.

For the purpose of this research we decided to use the stochastic kinetic model of neuron to solve the problem of noise reduction in an image. The whole problem was implemented with the help of `fsolve` Matlab function, which is based on the Lagrange multipliers method [7] of solving a set of non-linear equations.

The paper is organized as follows. In Section 2 we provide a brief description of the kinetic model of neuron in two versions, deterministic and stochastic. A full description of this model can be found e.g. in [8,9]. The problem of generalization is described in Section 3, and it is strictly based on the idea presented in [2]. In Section 4 we provide a description of the gradient descent algorithm of training for stochastic kinetic model of neuron, which is analogous to the training of the Hodgkin–Huxley model described in [10] and in similar form can be found in [11]. The training procedure is explained on the problem of noise reduction in an image. The results of noise reduction in an image for different forms of regularizer are discussed in Section 5. In the same section first-order optimality measure, root-mean-square error and histograms of performance tests are considered. Finally, in Section 6 a summary is provided.

2. Model description

We will consider a stochastic version of the kinetic model of neuron, which is an extension of the well-known Hodgkin–Huxley model of a neuron [12–15]. Between the neuron's interior and exterior it is possible to observe a difference of potential, which is called polarization. This difference of potential occurs as a result of uneven distribution of ions. In the interior of the neuron there are positive ions, while on the outside there are negative ions. The dynamics of the potential change on the membrane of the neuron is described with the following formula:

$$C\frac{dV}{dt} = I - g_{\mathrm{Na}}[m_3h_0]\cdot(V - V_{\mathrm{Na}}) + g_{\mathrm{K}}[n_4]\cdot(V - V_{\mathrm{K}}) + g_{\mathrm{L}}(V - V_{\mathrm{L}}). \tag{1}$$

In the above differential equation V stands for potential (output of the neuron) and I is the total membrane current density (input of the neuron) [12]. We assume that capacity per unit area C of the membrane is equal to 1 μF/cm^2. Parameters g_{Na}, g_{K}, g_{L} and V_{Na}, V_{K}, V_{L} are ion conductivities and ion potentials for sodium, potassium and chloride (the latter ones are associated with a small leakage current), respectively, and their values are shown in Table 1. Ions flow through the membrane thanks to ion channels, which are composed from smaller gates, from which the neuron's membrane is built.

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