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Finite element approximation of flow induced vibrations of human vocal folds model: Effects of inflow boundary conditions and the length of subglottal and supraglottal channel on phonation onset

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ABSTRACT

The paper presents the numerical analysis of interaction of the vibrating simplified human vocal folds model with the incompressible viscous airflow in a channel modeling simplified subglottal and supraglottal spaces. The flow in the considered 2D computational fluid domain is governed by the Navier–Stokes equations written in the Arbitrary Lagrangian–Eulerian form. The stabilized finite element method is applied for numerical approximation and the choice of boundary conditions and their implementation is discussed. For the considered model problem the prescribed inlet velocity and prescribed pressure difference formulations were numerically analyzed. The prescribed inlet velocity formulation was successful in predicting of the flutter velocity value, whereas the prescribed pressure difference gave nonphysical results. Finally a modified inlet boundary condition motivated by the penalization approach is suggested. It is shown that this approach gives possibilities to optimize the inlet boundary condition related to a physical reality by changing smoothly the penalty parameter in the interval between the two extremes and to treat the complete closures of the channel.

1. Introduction

Solution of fluid-structure interaction (FSI) problems is important in many technical applications, see e.g. [1,2]. Recently the FSI problems have become also important in biomechanics, see e.g. [3]. The mathematical simulation of FSI is a challenging problem. It is usually required to consider the viscous, frequently turbulent flow, changes of the flow domain in time and also the nonlinear behavior of the elastic structure. Moreover, the coupled system for the fluid flow and for the oscillating structure needs to be solved simultaneously. The changes of the fluid domain cannot be neglected and the methods with moving meshes or the immersed boundary methods must be employed, see e.g. [4,5].

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In modeling the problems of inner aero-hydroelasticity, i.e. when an elastic structure is a part of channel walls like for example vibrating vocal folds in human vocal tract or when some part of the channel walls is deformable, like e.g. blood flow in veins, it is usually difficult to set proper boundary conditions at the inlet and outlet of the channel.

For simplicity let's consider first the FSI problem of a solid rectangular flat plate with 2DOF (two degrees of freedom) for translation and rotation, and plainly installed on two springs in the wall of a channel conveying incompressible inviscid fluid modeled by a potential flow in 1D and 2D approximations, see [6,7]. When the inlet flow velocity was considered as the controlled parameter it was shown that the critical flutter velocity increased with the channel length of the outlet part situated downstream of the vibrating plate. If the pressure at the inlet of the channel was the controlled parameter, setting the pressure drop between the channel inlet and outlet, the critical flutter flow velocity for the loss of the system stability was strongly dependent both on the length of the inlet part of the channel situated upstream of the plate and on the length of the downstream part of the channel.

Similarly the results presented in the previous paper [8] of the authors showed that the flutter instability boundary of the aeroelastic system, modeling the flow-induced vibration of the human vocal folds situated in a channel between trachea and vocal tract, was substantially influenced by the type of prescribed boundary conditions at the inlet. For the inlet pressure boundary condition the system was stable up to very high unrealistic pressure drops between the inlet and outlet of the channel, in contrast to the case of the prescribed inlet velocity boundary condition when the flutter instability appeared in a realistic range of air flow rates.

Therefore the present paper is focused on a more detailed mathematical modeling of influence of the type of boundary conditions taking into account also the channel length at the inlet and outlet on the loss of stability of the aeroelastic system created by a simplified 2DOF model of the vocal folds self-excited by 2D incompressible viscous air flow with application to model the phonation onset.

For the medical reasons it is important to understand the basic principles of voice production and the mathematical models can be used to provide important information about the voicing, see e.g. [9]. However, the voice production is a complex mechanism which includes complicated airflow, vocal fold self-sustained vibrations and acoustics of human vocal tract airways. The primary source of human voice is created in the glottis - the narrow part of the channel between the two vocal folds. The vocal folds vibration leads to periodical (almost or complete) closure of the channel, which is the main source of the generated sound. The addressing of such a complex phenomenon encounters many difficulties.

Consequently several simplifications are usually made for the purpose of development of the computational models, like for example the two-mass dynamic model of the vocal folds [10] or similar lumped mass models using the same concept [11,12]. These simple mass-spring models coupled with a quasi-1D airflow were proved to be very useful. Such models can be used to estimate the vocal folds loading by impact stress during collisions, see [13]. The main advantage of these models is the ability to perform nearly real-time simulations.

However, the simplified flow models do not describe the details of the glottal airflow. The numerical solution of the 2D Navier–Stokes equations has been firstly used in [14], where the finite volume approximations of the airflow were coupled with the two-mass dynamic model. Later, 2D finite element (FE) approximations of the Navier–Stokes equations were coupled with low-dimensional model of vocal folds in [15]. The papers [16,17] can be considered as first articles modeling the fluid-structure interaction using the coupled FE models of the vocal folds and airflow. As mathematical models for the human phonation process are valuable tools for providing insight into the basic mechanisms of phonation many studies dealing with this FSI topic appeared in literature, see, e.g., [18–20] or [21]. However, the mathematical models usually do not address the complex problem in all details and use several simplifications.

The numerical analysis of 2D interaction of the incompressible flow with the vocal folds as has been published by the authors in [22] is considered also here. However, in this paper we focus on addressing the influence of the inlet and outlet lengths of the channel modeling the subglottal and supraglottal spaces on the flutter instability. Moreover, the applicability of the prescribed inlet/outlet boundary conditions in the case of almost complete closing of the channel due to the vocal folds vibrations is discussed.

The paper is structured as follows: First, the mathematical model addressing the fluid structure interactions is described. Further, the numerical approximation of the mathematical model by the finite element method is given. The numerical results are presented and the choice of boundary conditions and its implementation is discussed.

2. Mathematical model

The mathematical description of the interaction of incompressible flow and vocal folds consists of equations of motion for the vocal folds coupled with the incompressible Navier–Stokes equations via interface conditions, see Fig. 1. The fluid motion was modeled only in the lower half of the channel denoted by Ω_t^f and the assumption about the symmetry of the solution was made similarly as in [8]. First, the fluid flow in the computational domain surrounding the vibrating vocal fold is described. In order to take into account the time dependence of the computational fluid flow domain Ω_t^f, the so-called arbitrary Lagrangian–Eulerian (ALE) is used, see e.g. [23]. Further the motion of the structure is described by the simplified 2DOF model of the vocal folds for rotation and translation. Both models are coupled with the aid of interface conditions.

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