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A $Geo^{[X]}/G^{[X]}/1$ retrial queueing system with removal work and total renewal discipline

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ABSTRACT

In this paper we consider a discrete-time retrial queueing system with batch arrivals of geometric type and general batch services. The arriving group of customers can decide to go directly to the server expelling out of the system the batch of customers that is currently being served, if any, or to join the orbit. After a successful retrial all the customers in the orbit get service simultaneously. An extensive analysis of the model is carried out, and using a generating functions approach some performance measures of the model, such as the first distribution's moments of the number of customers in the orbit and in the system, are obtained. The generating functions of the sojourn time of a customer in the orbit and in the system are also given. Finally, in the section of conclusions and research results the main contributions of the paper are commented.

1. Introduction

There is a great potential for using the discrete-time queues in the performance analyses of computer and communication networks. The discrete-time queueing system has been found to be more appropriate in modeling computer and telecommunication systems than their continuous counterpart because the basic unit time in the discrete case is a binary code. Indeed, much of the usefulness of discrete-time queues derives from the fact that they can be used in the performance analysis of Digital Network and related computer communication technologies wherein the continuous-time models do not adapt [1,2].

Queueing systems with repeated attempts are characterized by the fact that a customer finding the server busy upon arrival must leave the service area and repeat its request for service after some random time. Between trials, the blocked customers join a group of unsatisfied customers called *orbit*. Retrial queues have been widely used to model many practical problems in telephone switching systems, telecommunication networks and computers competing to gain service from a central processing unit. For a detailed review of the main results and the literature on this topic the reader is referred to [3–8].

In many real telecommunication systems, it is frequently observed that the server processes the packets in groups. In such batch-service systems, jobs that arrive one at a time must wait in the queue until a sufficient number of jobs gets accumulated. A variety of batch-service queues with infinite waiting space has been studied by many researchers e.g. [9–13].

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This service discipline is closely related to other disciplines described in the queueing literature like G-networks, clearing systems, catastrophes, etc, see for example [14–18].

An interesting feature that is considered in this model is the total renewal discipline, that is, jobs or customers are served in groups (batch queues) but leaving the orbit empty at the moment of their batch service. Batch arrivals have been used to describe large deliveries and batch services to model a hospital out-patient department holding a clinic once a week, a transport link with capacity. Nowadays, home automation has greatly increased in popularity over the past several years, which refers to the automatic and electronic control of household features, activities, and appliances. Various control systems are utilized in this residential extension of building automation. The actions in this type of systems can be individual or in batches with total renewal discipline, for example, turning on the lights of a certain area at the same time when the alarm is turned on, combined with other actions. The automation of features in one's home helps to promote security, comfort, energy efficiency, and convenience.

Another feature that is usually found when a message is being processed in computers, in communications switching queues, etc, is that sometimes the information incoming to the server is more actual than the one on service. In that case, the message is moved to another place if the contained information can be used later on, or if the information is not any more valuable it is deleted, in both cases the server is upgraded. The mechanism of moving messages by the arrival of one of them is called synchronized or triggered motion. There are several mechanisms on how and where the messages are moved, for a survey on them refer to [14] and [19–21]. This mechanism of service interruptions was first studied in [22] that is considered an $M/M/1$ pre-emptive two-priority queueing model with exponentially distributed service interruption. An extensive study on such models can be consulted, for example, in [23–26] and for a detailed review on queues with service interruptions the reader is referred to [27].

The remainder of this paper is structured as follows. The assumptions of the queueing system under study are given in the next section. In Section 3 the Markov chain associated to our model is studied. The queue and system size distribution are obtained together with several performance measures of the system. In Section 4 the busy period (BP) is analysed and in Section 5 the sojourn times distribution of a customer in the queue and in the system with its respective means are given. Finally, a section of conclusions summarizes the main results for the system.

2. The mathematical model

In this paper a discrete-time queueing system in which the time axis is segmented into a sequence of equal intervals, called slots, is considered. It is assumed that all queueing activities (arrivals, departures and retrials) occur at the slot boundaries, and therefore, they may occur at the same time. So we suppose that the departures occur at the moment immediately before the slot boundaries, but external arrivals and retrials, in this order, occur at the moment after the slot boundaries.

Batches of customers arrive according to a geometrical arrival process with probability a, that is, a is the probability that an arrival occurs in a slot. The number of individual external customers arriving in each batch is $k, k \geq 1$, with probability c_k, and generating function (GF) $C(z) = \sum_{k=1}^{\infty} c_k z^k$, $0 < z \leq 1$. If an arriving batch of customers finds the server free, it begins immediately and jointly its service, otherwise, with probability θ it expels out of the system the group of customers that is currently being served, and starts immediately its service, or, with complementary probability $\bar{\theta}$ it joins the orbit in order to try its luck some time later.

The service times are independent and distributed with arbitrary distribution $\{s_i\}_{i=1}^{\infty}$, and generating function (GF) $S(x) = \sum_{i=1}^{\infty} s_i x^i$, $0 < x \leq 1$. Hence, s_i is the probability that a service lasts i slots. Let $S_k = \sum_{i=k}^{\infty} s_i$ denote the probability that the service lasts not less than k slots.

The retrials are jointly made by all the customers of the orbit. The retrial time (the time between two successive attempts) follows a geometrical law with probability $1 - r$, where r is the probability that the group of customers in the orbit does not make a retrial in a slot.

Once a service is finished, if no arrival occurs, and a successful retrial has taken place, all the customers of the orbit get service jointly and simultaneously.

3. The Markov chain associated to the system

At time k^+, the instant immediately after slot k, the state of the system can be described by the process $\{X_k,\ k \in \mathbb{N}\}$ with $X_k = (C_k, \xi_k, N_k^{(1)}, N_k^{(2)})$ where C_k denotes the state of the server 0 or 1 according to whether the server is free or busy, and $N_k^{(i)}$, $i = 1, 2$, is the number of customers in the server and in the orbit respectively. If $C_k = 1$, then ξ_k corresponds to the remaining service time of the group being served.

It can be shown that $\{X_k,\ k \in \mathbb{N}\}$ is the Markov chain of the queueing system under consideration, whose states space is

$$\{(0, n),\ n \geq 0;\ (i, m, n) : i, m \geq 1,\ n \geq 0\}.$$

Our first task is to find the stationary distribution:

$$\pi_{0,n} = \lim_{k \to \infty} P[C_k = 0,\ N_k^{(2)} = n],\ n \geq 0,$$