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New linear and quadratic prismatic piezoelectric solid–shell finite elements

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ABSTRACT

In this work, we propose two prismatic piezoelectric solid–shell elements based on fully three-dimensional kinematics. For this purpose, we perform electromechanical coupling, which consists in adding an electrical degree of freedom to each node of the purely mechanics-based versions of these elements. To increase efficiency, these geometrically three-dimensional elements are provided with some desirable shell features, such as a special direction, designated as the thickness, along which the integration points are located, while adopting a reduced integration rule in the other directions. To assess the performance of the proposed piezoelectric solid–shell elements, a variety of benchmark tests, both in static and vibration analysis, have been performed on multilayer structures ranging from simple beams to more complex structures involving geometric nonlinearities. Compared to conventional finite elements with the same kinematics, the evaluation results allow highlighting the higher performance of the newly developed solid–shell technology.

1. Introduction

In recent years, the so-called smart materials have aroused much interest in various fields and industrial applications. These smart materials and the associated devices are nowadays used in vibration control [1–3], shape control [4–6], noise and acoustic control [7–10] as well as in health monitoring of civil infrastructures [11–13]. Predicting the behavior of such materials and structures is therefore crucial for their proper implementation. For this purpose, the numerical simulation represents a very convenient and powerful approach, especially due to its very reasonable cost and its flexibility. Since the early work of Allik and Hughes [14], several tools have been proposed in the literature to model piezoelectric structures. Reviews on mechanical models and finite elements formulations, which can be found in [15–18], reveal that a significant number of 2D and 3D piezoelectric finite elements have actually been developed.

Robbins and Reddy [19] proposed an analysis of piezoelectrically actuated beams using a layer-wise displacement theory. Their work has been extended by Han and Lee [20] as well as by Hwang and Park [21], in order to analyze composite plates with piezoelectric actuators using 2D finite elements based on Kirchhoff's assumptions. Several other authors used First-order Shear Deformation Theory (FSDT) and Higher-order Shear Deformation Theory (HSDT), such as in [22–24] and in [25–28], respectively. To enrich the kinematics with respect to the above-discussed works, Kapuria et al. [29,30], Kapuria and Alam [31], and Kapuria and Kulkarni [32] introduced the well-known zigzag theory. Interesting contributions to the

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field were also made by Boudaoud et al. [33], Belouettar et al. [3] and Azrar et al. [34], among others, with applications to vibration control of multilayer structures. From a fundamental perspective, it is important to mention the major theoretical contributions of Weller and Licht [35,36], who studied the asymptotic behavior of thin piezoelectric plates (with or without electric field gradient). All of the above formulations are able to efficiently model beam and plate structures with piezoelectric materials. However, in real-life applications, it is common that relatively thin components coexist with thick structures, such as very thin piezoelectric patch sensors used for the monitoring of civil infrastructures. Consequently, the accurate and efficient modeling of such structures has motivated the development of new finite element technologies, among which the solid–shell concept. In this context also, several finite element models of this type have been proposed in the literature [37–42]. In particular, Sze and Yao [37], and Sze et al. [38] proposed hybrid finite element modeling of smart structures. In their work, the variation of electric potential was assumed to be linear along the thickness. Their formulation was later extended to the refined hybrid element by Zheng et al. [42]. Alternatively, Klinkel and Wagner [40,41] assumed in their contributions a quadratic distribution for the electric potential across the thickness. The geometric non-linearities were taken into account, but application of their model was restricted to structures combining elastic and piezoelectric layers. Tan and Vu-Quoc [39] also successfully modeled piezoelectric beam and plate structures under static and vibration conditions. More recently, Kulikov and Plotnikova [43,44] have developed solid–shell finite elements, which are like most of those developed in the literature, namely having a 2D geometry, while allowing a 3D constitutive law to be considered.

For the motivations described above, and for other well-known technological and practical requirements that have been widely discussed in the literature, the development of solid–shell elements based on three-dimensional kinematics is highly desirable. In this regard, we have recently contributed to this field by proposing linear and quadratic hexahedral piezoelectric solid–shell finite elements, denoted as SHB8PSE and SHB20E, respectively (see Kpeky [45]). These successful formulations of hexahedral piezoelectric solid–shell elements makes necessary the development of prismatic solid–shell elements, in order to easily and automatically model arbitrarily complex structures using free mesh generation tools.

In the current work, we propose to extend the prismatic linear and quadratic solid–shell elements SHB6 and SHB15, formulated in [46,47], respectively, on the basis of purely mechanical degrees of freedom, to the modeling of structures that contain piezoelectric materials. The remainder of the paper is organized as follows. In Section 2, the coupled electromechanical constitutive equations are presented as well as the discretized problem to be solved by the finite element method. Section 3 details the formulation of the SHB6E and SHB15E prismatic piezoelectric solid–shell elements, which are based on linear and quadratic interpolation, respectively. To assess the performance of the proposed piezoelectric solid–shell elements, a set of selective and representative benchmark tests are conducted in Section 4, both in static and vibration analysis. Finally, the main conclusions are summarized in Section 5.

2. Constitutive equations and discretization of the problem

2.1. Electromechanical constitutive equations

Piezoelectric materials have the capability of generating electricity when subjected to mechanical loading (sensors). Conversely, they also have the ability to deform under electrical charging (actuators). These properties are described by the following coupled electromechanical equations:

$$\begin{cases} \boldsymbol{\sigma} = \mathbf{C}\cdot\boldsymbol{\varepsilon} - \mathbf{e}^T\cdot\boldsymbol{E} \\ \boldsymbol{D} = \mathbf{e}\cdot\boldsymbol{\varepsilon} + \boldsymbol{\kappa}\cdot\boldsymbol{E} \end{cases} \tag{1}$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ represent, respectively, the vector form of the stress and strain tensors; $\boldsymbol{D}$ and $\boldsymbol{E}$ denote the electric displacement and electric field vector, respectively; while $\mathbf{C}$, $\mathbf{e}$ and $\boldsymbol{\kappa}$ stand for the elastic, piezoelectric and dielectric permittivity matrix, respectively.

The discretized forms $\{\boldsymbol{\varepsilon}\}$ and $\{\boldsymbol{E}\}$ for the strain tensor and the electric field vector are related, respectively, to the discretized displacement $\{\boldsymbol{u}\}$ and to the discretized electric potential $\{\boldsymbol{\phi}\}$, using the discrete gradient operators $[\mathbf{B}^u]$ and $[\mathbf{B}^\phi]$, as follows:

$$\begin{cases} \{\boldsymbol{\varepsilon}\} = [\mathbf{B}^u]\{\boldsymbol{u}\} \\ \{\boldsymbol{E}\} = -[\mathbf{B}^\phi]\{\boldsymbol{\phi}\} \end{cases} \tag{2}$$

In the current contribution, the discrete gradient operators $[\mathbf{B}^u]$ and $[\mathbf{B}^\phi]$ are obtained by finite element discretization for each of the proposed prismatic piezoelectric solid–shell formulations SHB6E and SHB15E, as will be detailed in Section 3.

2.2. Discretized problem

The variational principle pertaining to piezoelectric materials, which provides the governing equations for the associated boundary value problem, is described by the Hamilton principle [14]. In this weak form of equations of motion, the Lagrangian and the virtual work are appropriately adapted to include the electrical contributions, in addition to the more