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# Approximating solutions to a bilevel capacitated facility location problem with customer's patronization toward a list of preferences

Martha-Selene Casas-Ramírez[a], José-Fernando Camacho-Vallejo[a,*], Iris-Abril Martínez-Salazar[b]

[a] Universidad Autónoma de Nuevo León, Facultad de Ciencias Físico-Matemáticas, San Nicolás, Nuevo León, Mexico
[b] Universidad Autónoma de Nuevo León, Facultad de Ingeniería Mecánica y Eléctrica, San Nicolás, Nuevo León, Mexico

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This paper presents a bilevel capacitated facility location problem where customers are allocated to the facilities they patronize based on a predetermined list of preferences. The bilevel problem is composed of an upper level, where a company locates facilities to minimize locating and distributing costs; and a lower level, where customers aim to maximize their preferences by being allocated to the most preferred facilities to get their demands met. The complexity of the lower level problem, which is NP-hard, demands alternatives for obtaining, in general, the follower's rational reaction set. Hence, bilevel attainable solutions are defined for solving the bilevel problem in an efficient manner. Moreover, for obtaining valid bounds, a reformulation of the bilevel problem based on the lower level's linear relaxation is performed. Then, a cross entropy method is implemented for obtaining solutions in the upper level; while the lower level is solved in three different manners: by a greedy randomized adaptive procedure based on preferences, by the same procedure but based on a regret cost, and by an exact method (when possible). The conducted experimentation shows the competitiveness of the proposed algorithms, in terms of solution quality and consumed time, despite the complexity of the problem's components.



## 1. Introduction

Bilevel programming (BP) has been suitable for modeling situations where two non-cooperative decision-makers aim to optimize their own objective. For example, BP in logistics has helped to consider some involved parties in the specific supply chain that were not jointly considered before. In other words, instead of considering two interrelated processes in an independent way, BP is convenient for simultaneously considering both parties in a hierarchized manner during the decision-making processes [21].

In a general way, a bilevel programming problem can be approached from two perspectives: game theory and mathematical programming. From the game theory point of view, there are two competing decision-makers associated with the upper and lower level problems—hereafter, the leader and the follower, respectively. In that hierarchized sequential game, first the

* Corresponding author.
*E-mail addresses:* martha.casasrm@uanl.edu.mx (M.-S. Casas-Ramírez), jose.camachovl@uanl.edu.mx (J.-F. Camacho-Vallejo), iris.martinezslz@uanl.edu.mx (I.-A. Martínez-Salazar).

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leader establishes its strategy, then the follower rationally reacts by choosing its best strategy having complete knowledge of the leader's decision. That is, for a fixed leader's strategy $y$, the follower must select $x^*(y)$ according to his own objective. This problem is known as a non-cooperative Stackelberg game in which each party tries to maximize its payoffs. Also, it is assumed that perfect information is given since the leader will decide its strategy first, and, after that, the follower will decide his own strategy.

On the other hand, from the mathematical programming point of view, a BP problem consists of a mathematical programming problem in which the variables can be partitioned into two subsets. The main issue is that one of the subsets of variables must be given by the optimal solution of another mathematical programming problem; that is, a bilevel programming problem is a mathematical programming problem with another optimization problem in its constraints. To clarify the idea, the general formulation presented in [4] is shown. A bilevel programming problem can be given by min $\{F(x, y): \ y \in Y, G(x, y) \leq 0, x \in P(y)\}$ where $P(y) = \{x \in X : x \in \text{argmin}\{f(\hat{x}, y) : g(\hat{x}, y) \leq 0\}\}$. Constraint $x \in P(y)$ requires the follower's variables to correspond to the optimal solution of the lower level problem.

Both approaches have attracted the attention of researchers due to their applicability to real-life problems. For instance, interesting literature reviews regarding applications in planning, distribution, location, pricing, network design, humanitarian logistics, and environmental models, modeled as bilevel programming problems, appear in [4,12,21,34].

Location problems have been intensively studied under the BP scheme. For example, competitive facility location problems have a natural modeling with BP. In [20], the concept of competition between two firms for locating facilities is introduced. The problem was treated as a two stage process; in the first stage, both firms decide the location of the facilities, while in the second stage the firms choose the optimal prices for the offered products. Spatial market competition is introduced, differentiating that problem from others. The assumption, that the spatial market is a closed linear segment in which the customer's demand is uniformly distributed among the segment, is made. Also, inelastic demands, homogeneous products being offered at facilities, and both firms considering mill pricing, are expected. Therefore, customers will buy the cheaper products without regard to the distance from the facilities, and both firms will have the same cost function. As a result of that paper, the Hotelling's law was established, which states that both firms will cluster their facilities to the center of the market. Besides the discussion about the Stackelberg equilibrium, an analysis for some scenarios in which the Nash equilibrium does not exist is presented. Finally, different variations of the problem are mentioned, such as the optimization of a social function, and the existence of product transportation in multidimensional spaces, among others.

After Hotellings' contribution, many researchers were motivated to tackle these problems. In [14], a very complete taxonomy that involves many specific components of the competitive facility location problem is given. The proposed taxonomy contains the number of involved decision-makers, the price policy, the characteristics of the game (sequential, simultaneous, etc.), and the customer's behavior. For a detailed review of competitive facility location problems, the reader is referred to [15,22,27,30].

Under the bilevel discrete competitive location problem framework, but considering preferences for allocating customers to facilities, we can mention Beresnev [5]. There, two competing companies which successively will locate their facilities to maximize their profits were considered. Beresnev identifies the following three components in the problem: (1) a leader company who seeks to locate its facilities considering the follower's response, (2) a follower company who considers the facilities located by the leader and makes its decision to maximize its own market capture, and (3) a set of customers who are free to choose the facility that will meet their demand. The selected rule for allocating the customer to facilities is based on a pre-established list of preferences customers have regarding being supplied by the facilities. To solve the problem, upper bounds are computed through an auxiliary pseudo-Boolean function. Later, in [6], an extension of the previous work is presented. Cooperative and non-cooperative solutions for the same problem are presented. Also, the previous upper bounds are used to obtain initial solutions, which are improved by two local search algorithms. Numerical results show that better performance occurs when a generalized neighborhood is considered in the local search.

The customers' preferences have also been used for other bilevel location problems, such as the uncapacitated bilevel facility location problem. In this problem, a leader seeks to open facilities aiming to minimize the sum of locating and distributing costs, while a follower allocates customers to the most preferred facilities. The follower's decision will affect the distributing cost, modifying the leader's objective function. The customers' freedom to choose the facilities that will serve them adds more reality to the problem. In real life, customers choose facilities based on costs, preferences, a predetermined contract, or a loyalty coefficient, among others. Next, some papers devoted to location problems and considering customers' patronization of preferred facilities are discussed.

Integer single-level formulations for this problem had been proposed in the literature, where preferences are taken into account by including additional constraints. In fact, Hanjoul and Peeters [18] is the first paper in which a customer's ordered list of preferences (assuming an unknown number of facilities should be located—the simple plant location problem with order (SPLPO)) was considered. A greedy heuristic based on branch & bound was developed to solve the SPLPO. Numerical experiments on small-size instances (5 facilities and 8 customers) were conducted to validate the proposed solution method. Later, Ausiello et al. [3] demonstrated the facility location problem, with preferences of the customers and its variations, is an NP-hard problem. Also, in [9], new valid inequalities yielding tightened bounds for the integer problem are proposed. Through a pre-processing of the solutions, a reduction in the integrality gap is obtained in reasonable computational time.

Although the uncapacitated facility location problem with customer's preferences can be modeled as a single-level problem, an alternative and natural manner for formulating this is as a bilevel programming problem. Hansen et al. [19] is the seminal paper in which a bilevel model was developed for this problem. Bearing in mind that bilevel programming problems

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