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An implicit hybridized discontinuous Galerkin method for the 3D time-domain Maxwell equations

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ABSTRACT

We present a time-implicit hybridizable discontinuous Galerkin (HDG) method for numerically solving the system of three-dimensional (3D) time-domain Maxwell equations. This method can be seen as a fully implicit variant of classical so-called DGTD (Discontinuous Galerkin Time-Domain) methods that have been extensively studied during the last 10 years for the simulation of time-domain electromagnetic wave propagation. The proposed method has been implemented for dealing with general 3D problems discretized using unstructured tetrahedral meshes. We provide numerical results aiming at assessing its numerical convergence properties by considering a model problem on one hand, and its performance when applied to more realistic problems. We also include some performance comparisons with a centered flux time-implicit DGTD method.

1. Introduction

1.1. Generalities about the DGTD method

During the last ten years, the DGTD method has progressively emerged as a viable alternative to well established FDTD (Finite Difference Time-Domain) [1] and FETD (Finite Element Time-Domain) [2] methods for the numerical simulation of electromagnetic wave propagation problems in the time-domain.

The DGTD method can be considered as a finite element method where the continuity constraint at an element interface is released. While it keeps almost all the advantages of the finite element method (large spectrum of applications, complex geometries, etc.), the DGTD method has other nice properties which explain the renewed interest it gains in various domains in scientific computing:

- It is naturally adapted to a high order approximation of the unknown field. Moreover, one may increase the degree of the approximation in the whole mesh as easily as for spectral methods but, with a DGTD method, this can also be done locally i.e., at the mesh cell level. In most cases, the approximation relies on a polynomial interpolation method but the method also offers the flexibility of applying local approximation strategies that best fit to the intrinsic features of the modeled physical phenomena.
- When the discretization in space is coupled to an explicit time integration method, the DG method leads to a block diagonal mass matrix independently of the form of the local approximation (e.g., the type of polynomial interpolation).

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This is a striking difference with classical, continuous FETD formulations. Moreover, the mass matrix is diagonal if an orthogonal basis is chosen.

- It easily handles complex meshes. The grid may be a classical conforming finite element mesh, a non-conforming one or even a hybrid mesh made of various elements (tetrahedra, prisms, hexahedra, etc.). The DGTD method has been proven to work well with highly locally refined meshes. This property makes the DGTD method more suitable to the design of a *hp*-adaptive solution strategy (i.e., where the characteristic mesh size h and the interpolation degree p changes locally wherever it is needed).
- It is flexible with regards to the choice of the time stepping scheme. One may combine the discontinuous Galerkin spatial discretization with any global or local explicit time integration scheme, or even implicit, provided the resulting scheme is stable.
- It is naturally adapted to parallel computing. As long as an explicit time integration scheme is used, the DGTD method is easily parallelized. Moreover, the compact nature of method is in favor of high computation to communication ratio especially when the interpolation order is increased.

As in a classical finite element framework, a discontinuous Galerkin formulation relies on a weak form of the continuous problem at hand. However, due to the discontinuity of the global approximation, this variational formulation has to be defined at the element level. Then, a degree of freedom in the design of a discontinuous Galerkin scheme stems from the approximation of the boundary integral term resulting from the application of an integration by parts to the element-wise variational form. In the spirit of finite volume methods, the approximation of this boundary integral term calls for a numerical flux function which can be based on either a centered scheme or an upwind scheme, or a blend of these two schemes.

1.2. DGTD methods for time-domain electromagnetics

In the early 2000s, DGTD methods for time-domain electromagnetics have been studied by a few groups of researchers, most of them from the applied mathematics community. One of the most significant contributions is due to Hesthaven and Warburton [3] in the form of a high order nodal DGTD method formulated on unstructured simplicial meshes. The proposed formulation is based on an upwind numerical flux, nodal basis expansions on a triangle (2D case) and a tetrahedron (3D case) and a Runge–Kutta time stepping scheme. In [4], Kakbian et al. describe a rather similar approach. More precisely, the authors develop a parallel, unstructured, high order DGTD method based on simple monomial polynomials for spatial discretization, an upwind numerical flux and a fourth-order Runge–Kutta scheme for time marching. The method has been implemented with hexahedral and tetrahedral meshes. A high order DGTD method based on a strong stability preserving Runge–Kutta time scheme has been studied by Chen et al. [5]. The authors also present post-processing techniques that can double the convergence order. A locally divergence-free DGTD method is formulated and studied by Cockburn et al. in [6]. In the same period, a high order nodal DGTD method formulated on unstructured simplicial meshes has also been proposed by Fezoui et al. [7]. However, contrary to the DGTD methods discussed in [3,4], the DGTD method in [7] is non-dissipative thanks to a combination of a centered numerical flux with a second-order leap-frog time stepping scheme. The DGTD method has then been progressively considered and extended to increasingly more complex modeling situations by groups of researchers in the applied electromagnetics and electrical engineering communities for a wide variety of applications related to aeronautics, defense, semiconductor device fabrication, etc. [8–14] to cite a few. More recently, the method has also been adopted and further developed by researchers in the nano-optics domain [15–18]. A full review of the nowadays numerous applications of DGDT methods would certainly require than a simple paragraph. Also worth to note, the DGTD method has been implemented in commercial software such HFSS-TD (the time-domain version of the well-known HFSS software used for antenna design) [19].

1.3. Explicit versus implicit DGTD methods

From the above discussion, it is clear that the DGTD method is nowadays a very popular numerical method in the computational electromagnetics community. The works mentioned so far are mostly concerned with time explicit DGTD methods relying on the use of a single global time step computed so as to ensure stability of the simulation. It is however well known that when combined with an explicit time integration method and in the presence of an unstructured locally refine mesh, a high order DGTD method suffers from a severe time step size restriction. A possible alternative to overcome this limitation is to use smaller time steps, given by a local stability criterion, precisely where the smallest elements are located. The local character of a DG formulation is a very attractive feature for the development of explicit local time stepping schemes [20–22]. An alternative approach that has been considered in [23,24] is to use a hybrid explicit–implicit (or locally implicit) time integration strategy. Such a strategy relies on a component splitting deduced from a partitioning of the mesh cells in two sets respectively gathering coarse and fine elements. In these works, a second-order explicit leap-frog scheme is combined with a second-order implicit Crank–Nicolson scheme in the framework of a non-dissipative (centered flux based) DG discretization in space. At each time step, a large linear system must be solved whose structure is partly diagonal (for those rows of the system associated to the explicit unknowns) and partly sparse (for those rows of the system associated to the implicit unknowns). The computational efficiency of this locally implicit DGTD method depends on the size of the

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