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Riemann and Weierstrass walks revisited

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The Weierstrass and Riemann walks are non trivial discrete random processes to model and characterize the underlying "noise" in the dynamics of fluctuations for out of equilibrium systems, and, in more general contexts, to simulate complex dynamics like order-disorder phase transitions and anomalous diffusion properties in physical, biological and financial systems. In this work simple algorithms, implemented in GNU-R, for both Riemann and Weierstrass discrete processes are presented. Explicit formulas for the probability distributions of n steps are obtained. Finally a way to connect both random processes is commented.

1. Introduction

The continuous and discrete random processes represent an computationally attractive alternative choice to modeling non equilibrium complex systems such as financial time series [1–5], nucleation, aggregation and growth phenomena [6–8], propagation of inestabilities in ecological systems [9–13], coalescence and break-up dynamics [14–17], and in general, for modeling simple and anomalous diffusion processes[18–20].

Particularly, the Riemann walk is considered as the discrete version of the Lévy flight [21–24]. On the other hand, the Weierstrass random walk has been used for modeling the dynamics of anomalous diffusion processes [25], in the construction of a theoretical framework for relativistically covariant superdiffusive process [26] and, in its more generalized version, as Weierstrass–Mandelbrot process [27], and to simulate fractional Brownian motion [28].

The aim of this work is to compare the basic properties of the simple random walk with respect a couple of more general discrete random processes; to analyze its asymptotic properties and find explicit formulas to probability distributions of n steps, and give explicit algorithms cumulative distribution-based for the Riemann and Weierstrass walks. Moreover, to put in perspective the potential applications of this dicrete random processes to model time or data series with long-term or non-linear correlations.

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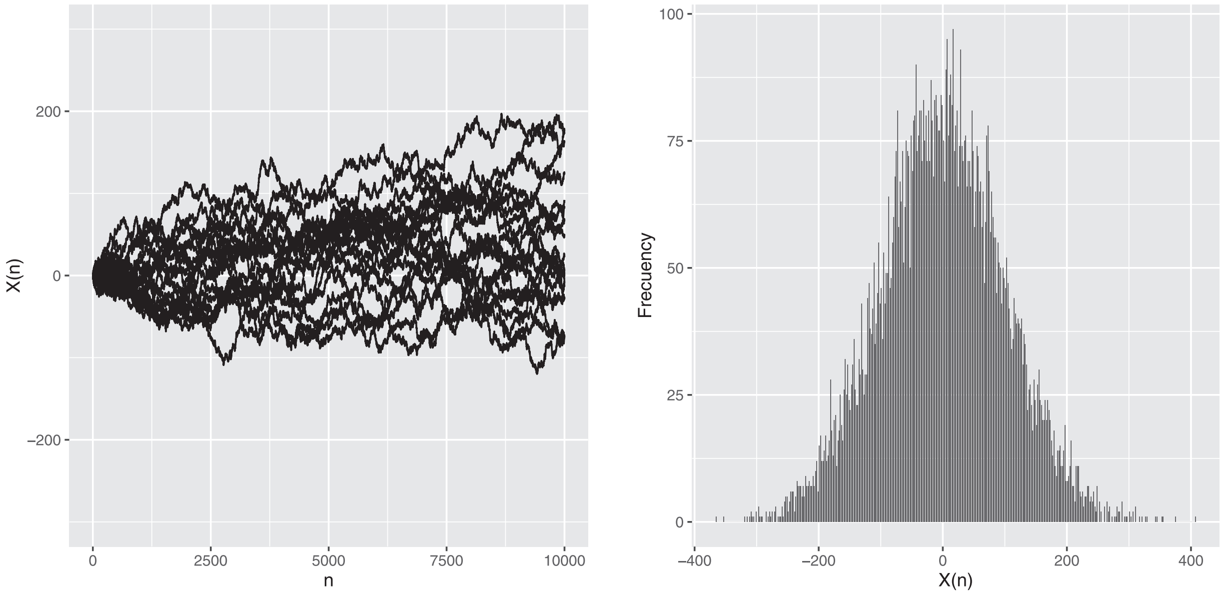

Fig. 1. Simple symmetric random walk. Left: 20 paths with $n = 10^4$ steps each one. Right: the histogram of 10^4 paths (final position X_n) with $n = 10^4$ steps each one. In this case, the empirical mean and standard deviation are $\bar{x} = -0.596$, $S = 100.7037$. The corresponding theoretical values are $\mu = E[X_n] = 0$ and $\sigma = \sqrt{Var(X_n)} = \sqrt{n} = 100$.

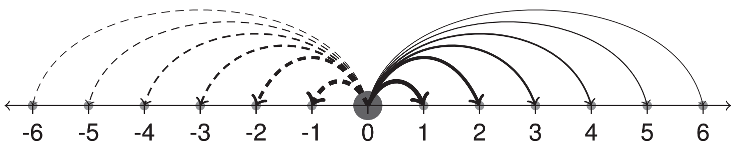

Fig. 2. Riemann random walk. Now, in contrast with the simple random walk, in each step a particle move to any point of an one-dimensional grid.

1.1. Simple random walk and Gaussian attractor

From a theoretical point of view, the simplest discrete random process to describe a non-equilibrium process is the simple random walk [29–32]. In this model a particle, initially in the origin of a one-dimensional grid, move one step to the right or left with probabilities p and $q = 1 - p$, respectively. Moreover, the steps are independent, i.e., if ξ_i represent the displacement in the ith step, then after n steps the resultant displacement X_n is the sum of n independent random variables: $X_n = \sum_{i=1}^{n} \xi_i$. Moreover, after n steps the probability of find to the particle in the position $X_n = j$ satisfies the binomial probability mass function

$$P_n(j) = \sum_{r=0}^{n} \frac{n!}{r!(n-r)!} p^r (1-p)^{n-r} \delta[j - (2r - n)] \tag{1}$$

where the argument of the delta means that the values r and j are not independent but they are related by the condition $j = 2r - n$, and the mean and variance are given by $E[X_n] = n(2p - 1)$ and $Var(X_n) = 4np(1 - p)$.

When $n \gg 1$, the DeMoivre–Laplace limit theorem [33] ensures that the discrete binomial distribution of the simple random walk can be approximated by the continuous normal distribution, see Fig. 1,

$$P_n(j) \sim \frac{1}{\sqrt{8\pi np(1-p)}} e^{-\frac{[j-n(2p-1)]^2}{8np(1-p)}} \tag{2}$$

2. Riemann random walk

An immediate generalization of the simple random walk is the Riemann random walk which the transition probabilities are no longer constant but it depend on the step size, i.e., $P(X_{n+1} = i + \ell \mid X_n = i) \equiv P(\xi = \ell)$, where $\xi = X_{n+1} - X_n$, see Fig. 2. Now, the step-size distribution is given by the Riemann probability mass function which is defined by the power-law, see Fig. 3, left side.

$$P(\xi = \ell) = \frac{c}{|\ \ell\ |^{\gamma+1}} \tag{3}$$