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Serial and parallel approaches for image segmentation by numerical minimization of a second-order functional

R. Zanella[a], F. Porta[a], V. Ruggiero[a,*], M. Zanetti[b]

[a] Mathematics and Computer Science Department, University of Ferrara, Ferrara, Italy
[b] Information and Communication Technology Department, University of Trento, Trento, Italy

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A B S T R A C T

Because of its attractive features, second order segmentation has shown to be a promising tool in remote sensing. A known drawback about its implementation is computational complexity, above all for large set of data. Recently in Zanetti et al. [1], an efficient version of the block-coordinate descent algorithm (BCDA) has been proposed for the minimization of a second order elliptic approximation of the Blake–Zissermann functional. Although the parallelization of linear algebra operations is expected to increase the performance of BCDA when addressing the segmentation of large-size gridded data (e.g., full-scene images or Digital Surface Models (DSMs)), numerical evidence shows that this is not sufficient to get significant reduction of computational time. Therefore a novel approach is proposed which exploits a decomposition technique of the image domain into tiles. The solution can be computed by applying BCDA on each tile in parallel way and combining the partial results corresponding to the different blocks of variables through a proper interconnection rule. We prove that this parallel method (OPARBCDA) generates a sequence of iterates which converges to a critical point of the functional on the level set devised by the starting point. Furthermore, we show that the parallel method can be efficiently implemented even in a commodity multicore CPU. Numerical results are provided to evaluate the efficiency of the parallel scheme on large images in terms of computational cost and its effectiveness with respect to the behavior on the tile junctions.

1. Introduction

Despite the wide image processing domain provides many different techniques to extract information from images, a major limitation which is often encountered is the capability to work in real application scenarios, e.g., in remote sensing (RS). As an example, digital surface models (DSMs) of the Earth's surface can be obtained from LiDAR (Light Ranging and Detection) point clouds. From this huge amount of data, useful information needs to be extracted systematically and, very often, this task must be accomplished on commodity machines with limited resources. Effective object-based or simplified representations of images can be obtained via segmentation approaches, where objects are identified by segments. To process large images, a common procedure consists in splitting the input into several tiles, running a specific segmentation algorithm separately on each tile and, then, merging together the partial results. However, this strategy is empirical in nature and the global dependency of the solution on data often introduces undesired artifacts which potentially propagate

* Corresponding author.
E-mail address: valeria.ruggiero@unife.it (V. Ruggiero).

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from the tiles junctions to their interior. It is a matter of fact that, the theoretical justification of tiling approaches for segmentation algorithms is rarely considered in literature.

Among the mathematical methods to address the image segmentation problem, a key role is played by variational models. In this case, the solution is theoretically formulated as a minimizer of a global energy. In the seminal paper [2], Mumford and Shah (MS) proposed a first-order functional, whose minimization determines an approximation of the image by means of a piecewise smooth function. The Blake–Zisserman (BZ) second-order model [3] has been introduced with the aim of overcoming the limitations of the Mumford–Shah approach, such as the over-segmentation of the step gradients and the lack in 2nd-order detection (gradient discontinuities) and the triple-point problem (see [4,5] for a theoretical study). Nevertheless, the original functional formulation of the segmentation problem by MS or BZ is too strong for a numerical treatment. Among many approaches employed to numerically compute minimizers, we recall the well-known Ambrosio–Tortorelli (AT) approximation of the MS functional [6], which is prone to be numerically implemented. In their functional model, Ambrosio–Tortorelli replaced the unknown discontinuity set by an auxiliary function which smoothly approximates its indicator function. The initial choice of the discontinuity function can be made in an energetically convenient manner by exploiting theoretical known properties of the solution. Moreover, the AT approximation of the MS functional enjoys partial (quadratic) convexity property; therefore gradient-based methods have in general satisfying performance. It is worth mentioning here that a multi-grid approach to speed-up computations is exploited in [7] and a parallel version of this approach is developed in [26]. The numerical treatment of the BZ functional is considered by Bellettini and Coscia [8,9] in the one dimensional case, and by Ambrosio, Faina, March in the two dimensional case [10]. In the latter, the main result is an elliptic approximation of the functional where the technique proposed by AT (to approximate the MS functional) has been properly adapted for the 2nd-order functional. Here, two auxiliary functions are introduced as indicators of both the discontinuity and gradient discontinuity sets of the solution. More specifically, let $\Omega \subset \mathbb{R}^2$ be an open set and $g \in L^\infty(\Omega)$ be a given image. The goal is to minimize the functional

$$\begin{aligned}\mathcal{F}_\epsilon(s,z,u) = \delta \int_\Omega z^2|\nabla^2 u|^2\,dx + \xi_\epsilon \int_\Omega (s^2+o_\epsilon)|\nabla u|^2\,dx + (\alpha-\beta)\int_\Omega \epsilon|\nabla s|^2 + \frac{1}{4\epsilon}(s-1)^2\,dx \\ +\beta\int_\Omega \epsilon|\nabla z|^2 + \frac{1}{4\epsilon}(z-1)^2\,dx + \mu\int_\Omega |u-g|^2\,dx, \qquad (1)\end{aligned}$$

in proper Sobolev spaces. Here δ, α, β, μ are strictly positive parameters ($2\beta \geq \alpha \geq \beta$) and the terms ξ_ϵ, o_ϵ are infinitesimals. The approximation of the BZ functional takes place when $\epsilon \to 0$. Notice that, in the limit case, the minimizing functions s and z must be 1 almost everywhere on Ω in order to keep the energy finite. From the numerical point of view, by fixing ϵ as a small value, we have that functions s, z are allowed to have variations from 1 to 0 in a small neighborhood of the jump and the crease set of u, as this inhibits the costly contribution of $|\nabla u|^2$ and $|\nabla^2 u|^2$, respectively. Recently, the numerical minimization of the nonconvex functional (1) has been obtained by an especially tailored version of a block-coordinate descent algorithm (BCDA) [11], based on a compact matrix formulation of the functional [1]. Although theoretical models such as MS and BZ are global, the non-convexity of the objective functionals forces numerical methods to provide sub-optimal solutions. The outcome of many numerical experiments has highlighted that, although the theoretical model is global, the solutions weakly depend on boundary conditions and they are energetically close to initial data. This fact motivated us in developing a tiling scheme to address the segmentation of large images where a minimizer of the functional is assembled by merging together local minimizers restricted to sub-portions of the image. Preliminary results were encouraging [12,13].

The aim of this work is to provide mathematical arguments which justify the heuristic of this procedure. We start from the consideration that a simple idea to deal with very large images might be to implement in a parallel way the linear algebra operations of BCDA. Nevertheless numerical evidence highlights that the gained performance is limited. Thus, inspired by recent papers on the convergence of descent scheme for semi-algebraic problems [14–16], we propose a parallel approach, based on the decomposition of the image into partially overlapping tiles, which enables us to obtain a satisfactory accuracy also on the strip regions around the tile boundaries. This parallel iterative scheme, hereafter denoted by OPARBCDA, is basically a descent method which independently computes a portion of the new iterate for any tile by BCDA. Convergence results for the sequence of iterates generated by OPARBCDA are obtained. Particularly, we prove that the whole sequence of the OPARBCDA iterates converges to a critical point of the objective function. A suitable parallel implementation of OPARDCDA, based on a run-time distribution of independent tasks on the available cores of a multicore commodity, enables to address the segmentation of large images. Further, few iterations of the scheme determines an approximate solution that shows similar accuracy in every subregions of the domain.

The paper is organized as follows: in Section 2, we describe the discretization of (1) and its features by introducing the notation. In Section 3, we recall the special version of the block-coordinate descent algorithm BCDA to address the numerical minimization of the discrete functional (1) and we give the convergence analysis of the scheme, by exploiting the Kurdyka–Łojasiewicz (KL) property of the polynomial discrete functional. In Section 4, we introduce a parallel method, named OPARBCDA and based on the decomposition of the image domain into overlapping tiles, we address its convergence properties; moreover, always in this section, after a description of the tools to parallelize the linear algebra operations of BCDA, we discuss how to exploit the intrinsic parallel feature of OPARBCDA in a multicore CPU. Finally, in Section 5, the results of a vast numerical experimentation enable to compare and evaluate the effectiveness on the proposed approaches.

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