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A reactive self-tuning scheme for multilevel graph partitioning

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ABSTRACT

We propose a new multilevel graph bi-partitioning approach (M-RRTS) using greedy construction and reactive-randomized tabu search (RRTS). RRTS builds upon local search by adding prohibitions (to enforce diversification) and self-tuning mechanisms to adapt meta-parameters in an online manner to the instance being solved. The novel M-RRTS approach adds a multi-scale structure to the previous method. The original graph is summarized through a hierarchy of coarser graphs. At each step, more densely-interconnected nodes at a given level of the hierarchy are coalesced together. The coarsest graph is then partitioned, and uncoarsening phases followed by refinement steps build solutions at finer levels until the original graph is partitioned. A variation of RRTS is applied for the refinement of partitions after each uncoarsening phase. We investigate various building blocks of the proposed multilevel scheme, such as different initial greedy constructions, different tie-breaking options and various matching mechanisms to build the coarser levels. Detailed experimental results are presented on the benchmark graphs from Walshaw's graph partitioning repository and potentially hard graphs. The proposed approach produces the record results for 14 of 34 graphs from the repository in lower CPU times with respect to competing approaches. These results confirm the value of the new self-tuning and multilevel strategy to rapidly adapt to new instances.

1. Introduction

Graphs are frequently used as abstractions when modelling application problems and cutting a graph into smaller pieces is one of the fundamental algorithmic operations [1]. Even if the final application concerns a different problem (such as graph traversal, finding paths, trees, and flows), partitioning large graphs is often an important sub-problem for complexity reduction or parallelism [1]. The partitioning problem arises in many areas of computer science, such as parallel processing, complex networks, road networks, image processing, sparse matrix factorization, network partitioning, and VLSI physical design [1,2].

The graph partitioning (GP) problem on a graph $G = (V, E)$ (V being the set of vertices and E the set of edges) consists of dividing vertices into disjoint subsets such that the number of edges whose endpoints are in different subsets is minimized. Equivalent terms are graph bisection, or graph bi-partitioning when the number of subsets is equal to two. A balanced

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bisection of a graph $G = (V, E)$ is an unordered pair (V_0, V_1) of subsets for graph V such that $V_0 \cup V_1 = V$ and $V_0 \cap V_1 = \emptyset$. An imbalance factor $\epsilon \geq 0$ can be added to permit certain degree of difference between cardinalities of k sets, by requiring that $|V_i| \leq (1+\epsilon)\lceil |V|/k \rceil\ \forall i \in \{1, \ldots, k\}$. If sets are *perfectly balanced* the difference between the cardinalities of two sets is as small as possible: zero if V contains an even number of vertices, one otherwise.

An edge $(i, j) \in E$ is *cut* by a bisection if its endpoints belong to different subsets. The objective to be minimized is the cut size, denoted as $f(V_0, V_1)$, the number of edges that are cut by the given partition.

We propose a novel multilevel graph bi-partitioning approach, called M-RRTS, which is based on reactive-randomized tabu search (RRTS) and initial greedy construction. It is based on simple self-tuning schemes that act during the algorithm run and require minimal user intervention. Differently from previous studies, M-RRTS exploits multilevel bi-partitioning strategy to improve the running time by taking advantage of fast greedy construction of initial partitions. To assess the success of the approach, we investigate various choices by using a diverse set of benchmark graphs. M-RRTS uses *Min-Max-Greedy* and *Differential Greedy* algorithms in the construction of initial partitions and reactive heuristics [2]. A greedy construction produces initial assignments in a short CPU time, therefore, independent repetitions of the greedy initialization can be used to choose better initial partitioning configurations. M-RRTS with different algorithms for the initial construction and with various move selection schemes in prohibition-based tabu search are studied by using Walshaw's benchmark graphs [3]. Additionally M-RRTS is compared to AMG [4] and MSS14 [5] algorithms using *potentially hard graphs* [4].

The rest of the paper is organized as follows. First the related work and RRTS are presented in Section 2. The new M-RRTS approach is introduced in Section 3. The experiments to assess the performance of M-RRTS are analysed in Section 4.

2. Related work for graph partitioning

A recent survey [1] presents recent trends and practical algorithms for balanced graph partitioning. Additionally there are studies that presents graph partitioning within the area of numerical analysis [6], introduces genetic algorithms and problem-specific issues [7], discusses heuristics and approximation algorithms used in the multilevel framework and focus mostly on coarsening by matching and local search by node-swapping heuristics [8], and describes geometric, combinatorial, spectral, and multilevel methods and their combination [9].

The seminal classical approach is Kernighan–Lin algorithm [10], which is used and extended by several studies. An important improvement is Fiduccia and Mattheyses [11] variant. There are also various methods for multilevel partitioning [12–16]. Geometric crossovers to deal with redundancy and feasibility issues of multiway graph partitioning problem is also investigated [17]. A multilevel algorithm for balanced partitioning is proposed, which integrates a powerful refinement procedure based on tabu search with periodic perturbations [18]. A multilevel memetic algorithm that integrates a new multi-parent crossover operator and tabu search is also proposed [19]. Another approach is proposed to partition graphs effectively especially for highly irregular structure and provides graph coarsening by iteratively contracting size-constrained clusterings that are computed with a label propagation algorithm [5]. The same algorithm that provides the size-constrained clusterings can also be used during uncoarsening as a fast and simple local search algorithm. Another proposal is a heuristic for detecting and moving clusters, which is based on a new population-based measure of the distance between vertices, which substantially improves the ability of a memetic algorithm to find good partitions [20]. Different matching and algebraic multigrid (AMG) based coarsening schemes are compared and experimented with the algebraic distance between nodes to demonstrate computational results on several classes of graphs [4].

The reactive and prohibition-based graph partitioning generalizes KL, the prohibition is chosen in a randomized and reactive (online self-tuning) way with a bias towards more successful choices in the previous part of the run [2]. The proposed RRTS method is summarized in the following subsection.

2.1. Reactive-randomized tabu search

Tabu search (TS) is a meta-heuristic based on local search, which adopts a simple *prohibition* mechanism to enforce diversification [21]. In the standard application of local search for graph partitioning, a move corresponds to relocating a vertex from V_0 to V_1 or *vice versa*. The configuration (i.e., solution candidate) is represented with a binary string, the bit associated to a vertex is 1 or 0, depending on the set it belongs to. A neighbouring configuration is obtained by changing a single bit in the binary string. There are several variations of prohibition-based search. In the simplest form of TS, a neighbour is prohibited if and only if it has been already visited during the previous part of the search. In FIXED_TS a prohibition parameter T is introduced to determine how long a move will remain prohibited after its execution [22].

Reactive Search Optimization [22] (RSO) deals with the issue of self-tuning meta-parameters, parameters which influence a specific algorithm like the prohibition period T. With self-tuning, the meta-parameters can be adapted not only to the specific optimization problem, but also to an individual instance being solved, and to the specific characteristics of a fitness surface in the vicinity of a current tentative solution.

RSO is based on simple self-tuning schemes, acting while the algorithm runs: the value of the meta-parameters depends on the previous history of the local-search process. More general usages of machine learning schemes in optimization heuristics are studied in [23].