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Analysis of a full discretization scheme for $2D$ radiative-conductive heat transfer systems

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Abstract

This paper deals with the convergence of numerical scheme for combined nonlinear radiation-conduction heat transfer system in a gray, absorbing and non-scattering two-dimensional medium. The radiative transfer equation is solved using a Discontinuous Galerkin method with upwind fluxes. The conductive equation is discretized using the finite element method. Moreover, the Crank-Nicolson scheme is applied for time discretization of the semi-discrete nonlinear coupled system. Existence and uniqueness of the solution for the continuous and full discrete system are presented. The convergence proof follows from the application of a discrete fixed-point theorem, involving only the temperature fields at each time step. The order of approximation error, stability, and order of convergence are investigated. Finally, the theoretical stability and convergence results are supported with numerical examples.

Keywords: Radiative-conductive heat transfer, Galerkin method, Crank-Nicolson scheme, Banach fixed point theorem, error estimates.

Introduction

The aim of the present paper is to analyze a numerical method for nonlinear partial differential equations of radiative-conductive heat transfer system. The model considers a radiative transfer equation (RTE) coupled with a nonlinear conductive heat transfer equation (CE) in two-dimensional cases, i.e., the projection of the surface on the unit sphere in $\mathbb{R}^3$ onto the plane of the cross-section of the cylinder.Here, we present a proof of the existence and uniqueness of the solution for the full discrete numerical scheme. This discrete system is obtained by coupling discontinuous Galerkin (DG) numerical method applied to the RTE and finite element method (FEM) applied to the CE. Theoretical results about the stability, the convergence of the algorithm and the error estimates are presented. We previously proved the existence and uniqueness of the solution of the considered PDE system, see [1]. Moreover, a large number of numerical results has been presented in Ghattassi et al. [2]. In fact, the choice of DG methods is

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