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# Superconvergence of a modified weak Galerkin approximation for second order elliptic problems by $L^2$ projection method

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## Abstract

This paper derives a superconvergence result for the modified weak Galerkin (MWG) finite element method of the second order elliptic problem. The convergence rate of the MWG approximation is improved by 30% after applying a low cost  $L^2$  projection post-processing technique. These superconvergence phenomena are proved theoretically and confirmed numerically.

*Keywords:* Weak Galerkin FEM, Modified weak Galerkin, Finite element method, Superconvergence,  $L^2$ -projection method

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## 1. Introduction

The superconvergence of the finite element method is a widely known phenomenon where a new approximation constructed by post processing techniques is closer to the exact solution than the finite element solution with very little additional computation. The main idea of  $L^2$  projection method is to replace the finite element solution by its image under the  $L^2$ -projection from the finite element space onto another finite element space with a coarser mesh. The goal of this paper is to obtain theoretical results for the superconvergence of the modified weak Galerkin finite element approximation by  $L^2$ -projection method and to perform numerical experiments which support the theoretical findings. As our model problem, we consider the second order elliptic problem with a homogeneous boundary condition which seeks  $u \in H_0^1(\Omega)$  with

$$-\Delta u = f \quad \text{in } \Omega, \quad (1.1)$$

$$u = 0 \quad \text{on } \partial\Omega, \quad (1.2)$$

where  $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$  is the Laplacian operator,  $\Omega$  is an open, bounded, and polytopal domain in  $\mathbb{R}^2$  with Lipschitz continuous boundary  $\partial\Omega$ . We use the standard definitions for Sobolev spaces  $H^s(\Omega)$  and their associated inner products  $(\cdot, \cdot)_{s,\Omega}$ , norms  $\|\cdot\|_{s,\Omega}$ , and seminorms  $|\cdot|_{s,\Omega}$ ,  $s \geq 0$ . The space  $H^0(\Omega)$  coincides with  $L^2(\Omega)$ , in which case the norm and inner product are denoted by  $\|\cdot\|_\Omega$  and  $(\cdot, \cdot)_\Omega$ , respectively.

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