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Superconvergence of a modified weak Galerkin approximation for second order elliptic problems by L^2 projection method

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Abstract

This paper derives a superconvergence result for the modified weak Galerkin (MWG) finite element method of the second order elliptic problem. The convergence rate of the MWG approximation is improved by 30% after applying a low cost L^2 projection post-processing technique. These superconvergence phenomena are proved theoretically and confirmed numerically.

Keywords: Weak Galerkin FEM, Modified weak Galerkin, Finite element method, Superconvergence, L^2-projection method

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1. Introduction

The superconvergence of the finite element method is a widely known phenomenon where a new approximation constructed by post processing techniques is closer to the exact solution than the finite element solution with very little additional computation. The main idea of L^2 projection method is to replace the finite element solution by its image under the L^2-projection from the finite element space onto another finite element space with a coarser mesh. The goal of this paper is to obtain theoretical results for the superconvergence of the modified weak Galerkin finite element approximation by L^2-projection method and to perform numerical experiments which support the theoretical findings. As our model problem, we consider the second order elliptic problem with a homogeneous boundary condition which seeks $u \in H_0^1(\Omega)$ with

$$-\Delta u = f \quad \text{in} \quad \Omega, \tag{1.1}$$

$$u = 0 \quad \text{on} \quad \partial\Omega, \tag{1.2}$$

where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplacian operator, Ω is an open, bounded, and polytopal domain in $\mathbb{R}^2$ with Lipschitz continuous boundary $\partial\Omega$. We use the standard definitions for Sobolev spaces $H^s(\Omega)$ and their associated inner products $(\cdot,\cdot)_{s,\Omega}$, norms $\|\cdot\|_{s,\Omega}$, and seminorms $|\cdot|_{s,\Omega}$, $s \geqslant 0$. The space $H^0(\Omega)$ coincides with $L^2(\Omega)$, in which case the norm and inner product are denoted by $\|\cdot\|_\Omega$ and $(\cdot,\cdot)_\Omega$, respectively.

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