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Highly efficient family of iterative methods for solving nonlinear models☆

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Abstract

In this study, we present a new highly efficient sixth-order family of iterative methods for solving nonlinear equations along with convergence properties. Further, we extend this family to the multidimensional case preserving the same order of convergence. In order to illustrate the applicability of our methods, we choose some real world problems namely, kinematic syntheses, boundary value, Bratu's 2D, Fisher's and Hammerstein integral problems in the case of nonlinear systems (see [23, 29] for more applications) . In addition, numerical comparisons are made to show the performance of our iterative techniques with the existing ones. Moreover, we find that our techniques perform better as compared to the existing ones of same order in terms of residual error and the errors between two consecutive iterations. Finally, the stability analysis of our methods also support this to great extent.

Keywords: Nonlinear systems, iterative methods, convergence, basin of attraction, parameter plane, stability.

1. Introduction

In this paper, we consider

$$F(x) = 0, \tag{1.1}$$

where $F : \mathbb{I} \subset \mathbb{R}^n \to \mathbb{R}^n$ is a univariate function when $n = 1$ or multivariate function when $n > 1$ on an open domain $\mathbb{I}$.

Construction of higher-order iterative methods in order to approximate the solutions of (1.1) is one of the most important and challenging task in the field of numerical analysis. The importance of this subject led to the development of many numerical techniques. However, most of them are iterative in nature because analytic methods for such problems are almost non existent. Therefore, scholars from the worldwide are trying their best to resort an iterative method from the past few decades. In addition, iterative methods provide an approximated solution corrected up to a specified degree of accuracy which is further depend upon the considered iterative method and programming software namely, Fortran, Maple, Matlab, Mathematica, etc. Moreover, researchers have to face several problems while using them. Some of them are related to slower convergence, non-convergence, oscillation problem close to the initial guess, divergence, failure etc. (for the detail explanation of these problems please see [35, 39]).

There are several examples where we can see the applicability of them to real world problems. For example, Moré [33] proposed a collection of nonlinear model problems and most of them are phrased in the terms of $F(x) = 0$. On the other hand, Grosan and Abraham [25], also discussed the applicability of the nonlinear systems in neurophysiology, kinematics syntheses, chemical equilibrium, combustion and economics modeling problems. In addition, the reactor and steering problems were solved in [17, 40] by phrasing them in the form of $F(x) = 0$. Moreover, Lin et al. [31] also discussed the applicability of the nonlinear system in transportation theory.

In the past and recent years, researchers proposed a plethora of one point and multi-point methods for obtaining the solutions of (1.1) (we can see an overview in [14, 15, 35, 39]). Most of them are the extension of the classical Newton or Newton like method at the expense of some additional functional evaluations.

There are two main ways to construct higher-order iterative methods for nonlinear systems. In the first way, scholars proposed new iterative methods for scalar equation. Then, they extended the same scheme to the multidimensional case

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